3.5

9. $\lim_{x \to 1^-} \frac{1+x}{1-x} = \infty$ since the numerator approaches 2 and the denominator approaches 0 through positive values as $x \to 1$ from the left.

15. $\lim_{x \to -2^+} \left(\frac{1}{x+3} - \frac{x}{x+2} \right) = \infty$. As $x \to -2$ from the right, the first term approaches 1 but the second term approaches $-\infty$.

25. $$\lim_{x\to\infty} \left(\frac{x^3}{3x^2-2} - \frac{x^2}{3x+1} \right) = \lim_{x\to\infty} \left[\frac{x^3(3x+1) - x^2(3x^2-2)}{(3x^2-2)(3x+1)} \right] = \lim_{x\to\infty} \frac{x^3+2x^2}{(3x^2-2)(3x+1)}$$
$$= \lim_{x\to\infty} \frac{1+\frac{2}{x}}{\left(3-\frac{2}{x^2}\right)\left(3+\frac{1}{x}\right)} = \frac{1}{9}$$

35. $\lim_{x\to\infty} x \sin \frac{1}{x} = \lim_{x\to\infty} \frac{\sin \frac{1}{x}}{\frac{1}{x}}$. Letting $h = \frac{1}{x}$, this is equal to $\lim_{h\to 0} \frac{\sin h}{h} = 1$.

72. $\lim_{x\to\infty} \left(\frac{2x^2+3}{x+1} - ax - b \right) = \lim_{x\to\infty} \left[\frac{2x^2+3-(x+1)(ax+b)}{x+1} \right] = \lim_{x\to\infty} \frac{(2-a)x^2-(a+b)x+3-b}{x+1}$.

If the limit is 0, then $2 - a = 0 \Rightarrow a = 2$. Then the limit reduces to

$\lim_{x\to\infty} \frac{-(2+b)x+3-b}{x+1} = \lim_{x\to\infty} \frac{-(2+b)+\frac{3-b}{x}}{1+\frac{1}{x}} = -(2+b)$. Setting $-(2+b) = 0$ gives $b = -2$.

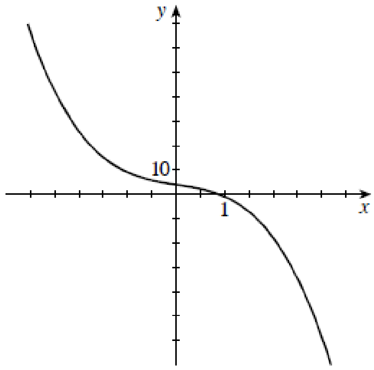
3. 6

5. $g(x) = 4 - 3x - 2x^3$

(1) The domain of f is $(-\infty, \infty)$. **(2)** Setting $x = 0$ gives $y = 4$ as the y-intercept. Setting $y = g(x) = 0$ gives a cubic equation which is not easily solved, so we will not attempt to find the x-intercept. **(3)** There is no symmetry. **(4)** $\lim_{x\to-\infty} g(x) = \infty$ and $\lim_{x\to\infty} g(x) = -\infty$.

(5) There is no asymptote. **(6)** $g'(x) = -3 - 6x^2 = -3\left(2x^2 + 1\right) < 0$ for all values of x, and so g is decreasing on $(-\infty, \infty)$. **(7)** The preceding step shows that g has no critical number and hence has no relative extremum. **(8)** $g''(x) = -12x$. Since $g''(x) > 0$ for $x < 0$ and $g''(x) < 0$ for $x > 0$, we see that g is concave upward on $(-\infty, 0)$ and concave downward on $(0, \infty)$. **(9)** From step 8, we see that $(0, 4)$ is an inflection point of g.

(10)

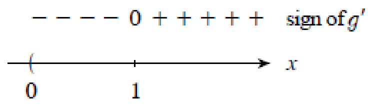


14. $g(x) = \frac{1}{2}x - \sqrt{x}$

(1) The domain of g is $[0, \infty)$. **(2)** The y-intercept is 0. To find the x-intercept, set $y = 0 \Leftrightarrow \frac{1}{2}x - \sqrt{x} = 0 \Leftrightarrow x = 2\sqrt{x} \Leftrightarrow x^2 = 4x \Leftrightarrow x(x-4) = 0 \Leftrightarrow x = 0$ or $x = 4$.

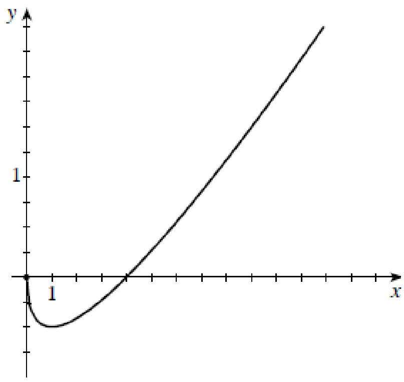
(3) $\lim_{x\to\infty}\left(\frac{1}{2}x - \sqrt{x}\right) = \lim_{x\to\infty}\frac{1}{2}x\left(1 - \frac{2}{\sqrt{x}}\right) = \infty$ **(4)** There is no symmetry. **(5)** There are no asymptotes.

(6) $g'(x) = \frac{1}{2} - \frac{1}{2}x^{-1/2} = \frac{1}{2}x^{-1/2}\left(x^{1/2} - 1\right) = \dfrac{\sqrt{x} - 1}{2\sqrt{x}} = 0 \Leftrightarrow x = 1$. From the sign diagram for g', we see that g is decreasing on $(0, 1)$ and increasing on $(1, \infty)$. **(7)** From the sign diagram of g', we see that $g(1) = -\frac{1}{2}$ is a relative minimum.

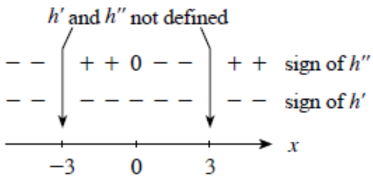


(8) $g''(x) = \left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)x^{-3/2} = \dfrac{1}{4x^{3/2}} > 0$ for $x > 0$, and so g is concave upward on $(0, \infty)$. **(9)** There is no inflection point.

(10)



19. $h(x) = \dfrac{x}{x^2-9}$

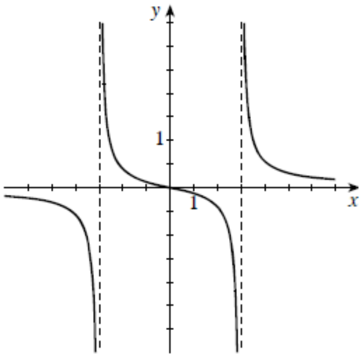


(1) The domain of h is $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$. **(2)** The x- and y-intercepts are 0. **(3)** $h(-x) = \dfrac{-x}{(-x)^2-9} = -\dfrac{x}{x^2-9} = -h(x)$, so the graph of h is symmetric with respect to the origin.

(4) $\lim\limits_{x\to-\infty} h(x) = 0$ and $\lim\limits_{x\to\infty} h(x) = 0$. **(5)** From (4), we see that $y=0$ is a horizontal asymptote. Since $\lim\limits_{x\to-3^-} h(x) = \lim\limits_{x\to3^-} h(x) = -\infty$ and $\lim\limits_{x\to-3^+} h(x) = \lim\limits_{x\to3^+} h(x) = \infty$, $x=\pm3$ are vertical asymptotes.

(6) $h'(x) = \dfrac{\left(x^2-9\right) - x(2x)}{\left(x^2-9\right)^2} = -\dfrac{x^2+9}{\left(x^2-9\right)^2}$. From the sign diagram for h' we see that h is decreasing on its domain. **(7)** f has no relative extremum.

(10)



(8) $h''(x) = -\dfrac{\left(x^2-9\right)^2(2x) - \left(x^2+9\right)2\left(x^2-9\right)(2x)}{\left(x^2-9\right)^4} = \dfrac{2x\left(x^2+27\right)}{\left(x^2-9\right)^3}$. From the sign diagram of h'', we see that h is concave downward on $(-\infty,-3)$ and $(0,3)$ and concave upward on $(-3,0)$ and $(3,\infty)$. **(9)** h has an inflection point at $(0,0)$. Neither of ±3 is in the domain of h.

33. $f(x) = \dfrac{x^2-2x-3}{2x-2}$.

$$\lim_{x\to\infty}\frac{f(x)}{x} = \lim_{x\to\infty}\frac{x-2-3/x}{2x-2} = \lim_{x\to\infty}\frac{1-2/x-3/x^2}{2-2/x} = \frac{1}{2} = m$$

and

$$\lim_{x\to\infty}\left[f(x)-mx\right] = \lim_{x\to\infty}\left(\frac{x^2-2x-3}{2x-2} - \frac{x}{2}\right) = \lim_{x\to\infty}\frac{-x-3}{2x-2}$$

$$= -\tfrac{1}{2} = b, \text{ so } y = \tfrac{1}{2}x - \tfrac{1}{2} \text{ is a slant asymptote.}$$

Using the curve sketching guidelines, we obtain the graph at right.