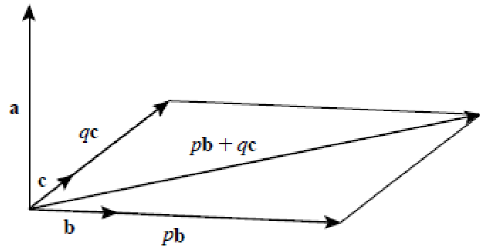
11.3

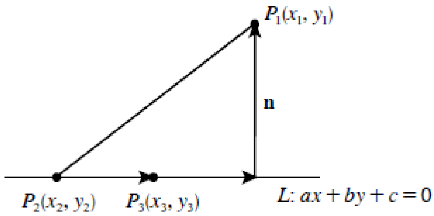
60. $\mathbf{a}$ is orthogonal to $\mathbf{b} \Rightarrow \mathbf{a} \cdot \mathbf{b} = 0$ and $\mathbf{a}$ is orthogonal to $\mathbf{c} \Rightarrow \mathbf{a} \cdot \mathbf{c} = 0$. Thus, $\mathbf{a} \cdot (p\mathbf{b} + q\mathbf{c}) = \mathbf{a} \cdot (p\mathbf{b}) + \mathbf{a} \cdot (q\mathbf{c}) = p\,(\mathbf{a} \cdot \mathbf{b}) + q\,(\mathbf{a} \cdot \mathbf{c}) = 0$, showing that $\mathbf{a}$ is orthogonal to $p\mathbf{b} + q\mathbf{c}$. To interpret the result geometrically, observe that the vector $p\mathbf{b} + q\mathbf{c}$ lies in the plane determined by $\mathbf{b}$ and $\mathbf{c}$. Since $\mathbf{a}$ is orthogonal to both $\mathbf{b}$ and $\mathbf{c}$, it is orthogonal to any vector lying in the plane determined by $\mathbf{b}$ and $\mathbf{c}$.



63. **a.** Pick $P_3(x_3, y_3)$ to be a point on L distinct from $P_2(x_2, y_2)$ and consider

$$\mathbf{n} \cdot \overrightarrow{P_2P_3} = \langle a, b\rangle \cdot \langle x_3 - x_2, y_3 - y_2\rangle = a\,(x_3 - x_2) + b\,(y_3 - y_2)$$
$$= (ax_3 + by_3) - (ax_2 + by_2)\ (1)$$

Since P_2 and P_3 both lie on L, we have $ax_2 + by_2 = -c$ and $ax_3 + by_3 = -c$. Substituting into (1) gives $\mathbf{n} \cdot \overrightarrow{P_2P_3} = -c - (-c) = 0$, so $\mathbf{n}$ is orthogonal to L.



b. $d = \dfrac{\left|\overrightarrow{P_1P_2} \cdot \mathbf{n}\right|}{|\mathbf{n}|} = \dfrac{|\langle x_2 - x_1, y_2 - y_1\rangle \cdot \langle a, b\rangle|}{\sqrt{a^2 + b^2}} = \dfrac{|a\,(x_2 - x_1) + b\,(y_2 - y_1)|}{\sqrt{a^2 + b^2}}$. Since $P_2(x_2, y_2)$ lies on L, $ax_2 + by_2 = -c$, so $d = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

c. Here $P_1(1, -3)$ and $L : 2x + 3y - 6 = 0$, so $d = \dfrac{|2\,(1) + 3\,(-3) - 6|}{\sqrt{4 + 9}} = \dfrac{13}{\sqrt{13}} = \sqrt{13}$.

11.4

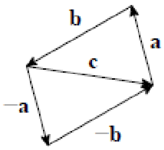
43. $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})+\mathbf{b}\times(\mathbf{c}\times\mathbf{a})+\mathbf{c}\times(\mathbf{a}\times\mathbf{b})=(\mathbf{a}\cdot\mathbf{c})\,\mathbf{b}-(\mathbf{a}\cdot\mathbf{b})\,\mathbf{c}+(\mathbf{b}\cdot\mathbf{a})\,\mathbf{c}-(\mathbf{b}\cdot\mathbf{c})\,\mathbf{a}+(\mathbf{c}\cdot\mathbf{b})\,\mathbf{a}-(\mathbf{c}\cdot\mathbf{a})\,\mathbf{b}$

$$=(\mathbf{a}\cdot\mathbf{c})\,\mathbf{b}-(\mathbf{a}\cdot\mathbf{b})\,\mathbf{c}+(\mathbf{a}\cdot\mathbf{b})\,\mathbf{c}-(\mathbf{b}\cdot\mathbf{c})\,\mathbf{a}+(\mathbf{b}\cdot\mathbf{c})\,\mathbf{a}-(\mathbf{a}\cdot\mathbf{c})\,\mathbf{b}=\mathbf{0}$$

44. $(\mathbf{a}\times\mathbf{b})\cdot(\mathbf{c}\times\mathbf{d})=[(\mathbf{a}\times\mathbf{b})\times\mathbf{c}]\cdot\mathbf{d}=[-\mathbf{c}\times(\mathbf{a}\times\mathbf{b})]\cdot\mathbf{d}=-[(\mathbf{c}\cdot\mathbf{b})\,\mathbf{a}-(\mathbf{c}\cdot\mathbf{a})\,\mathbf{b}]\cdot\mathbf{d}$

$$=(\mathbf{c}\cdot\mathbf{a})(\mathbf{b}\cdot\mathbf{d})-(\mathbf{c}\cdot\mathbf{b})(\mathbf{a}\cdot\mathbf{d})=(\mathbf{a}\cdot\mathbf{c})(\mathbf{b}\cdot\mathbf{d})-(\mathbf{b}\cdot\mathbf{c})(\mathbf{a}\cdot\mathbf{d})=\begin{vmatrix}\mathbf{a}\cdot\mathbf{c} & \mathbf{b}\cdot\mathbf{c}\\ \mathbf{a}\cdot\mathbf{d} & \mathbf{b}\cdot\mathbf{d}\end{vmatrix}$$

46. a. By the parallelogram law of vector addition, we have $-\mathbf{a}+(-\mathbf{b})=\mathbf{c}\Leftrightarrow$ $\mathbf{a}+\mathbf{b}+\mathbf{c}=\mathbf{0}$ as in the figure.



b. $\mathbf{a}\times(\mathbf{a}+\mathbf{b}+\mathbf{c})=\mathbf{a}\times\mathbf{0}\Rightarrow\mathbf{a}\times\mathbf{b}+\mathbf{a}\times\mathbf{c}=\mathbf{0}$ because $\mathbf{a}\times\mathbf{a}=0$, and $\mathbf{c}\times(\mathbf{a}+\mathbf{b}+\mathbf{c})=\mathbf{c}\times\mathbf{0}\Rightarrow\mathbf{c}\times\mathbf{a}+\mathbf{c}\times\mathbf{b}=\mathbf{0}\Rightarrow$ $-\mathbf{a}\times\mathbf{c}-\mathbf{b}\times\mathbf{c}=0$. Thus, $\mathbf{a}\times\mathbf{b}-\mathbf{b}\times\mathbf{c}=\mathbf{0}\Rightarrow\mathbf{a}\times\mathbf{b}=\mathbf{b}\times\mathbf{c}$. Next, $\mathbf{b}\times(\mathbf{a}+\mathbf{b}+\mathbf{c})=\mathbf{0}\Rightarrow\mathbf{b}\times\mathbf{a}+\mathbf{b}\times\mathbf{c}=\mathbf{0}$, so $\mathbf{b}\times\mathbf{c}=\mathbf{c}\times\mathbf{a}$. Thus, $\mathbf{a}\times\mathbf{b}=\mathbf{b}\times\mathbf{c}=\mathbf{c}\times\mathbf{a}\Rightarrow|\mathbf{a}\times\mathbf{b}|=|\mathbf{b}\times\mathbf{c}|=|\mathbf{c}\times\mathbf{a}|\Rightarrow|\mathbf{a}|\,|\mathbf{b}|\sin C=|\mathbf{b}|\,|\mathbf{c}|\sin A=|\mathbf{c}|\,|\mathbf{a}|\sin B$, and dividing by $|\mathbf{a}|\,|\mathbf{b}|\,|\mathbf{c}|$ yields $\dfrac{\sin C}{c}=\dfrac{\sin A}{a}=\dfrac{\sin B}{b}$, where $a=|\mathbf{a}|$, $b=|\mathbf{b}|$, and $c=|\mathbf{c}|$.

52. Let $\mathbf{a}=a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k}$, $\mathbf{b}=b_1\mathbf{i}+b_2\mathbf{j}+b_3\mathbf{k}$, and $\mathbf{c}=c_1\mathbf{i}+c_2\mathbf{j}+c_3\mathbf{k}$. Then

$$\mathbf{b}\times\mathbf{c}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}=(b_2c_3-b_3c_2)\,\mathbf{i}-(b_1c_3-b_3c_1)\,\mathbf{j}+(b_1c_2-b_2c_1)\,\mathbf{k}\text{ and}$$

$$\mathbf{a}\times(\mathbf{b}\times\mathbf{c})=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ a_1 & a_2 & a_3\\ b_2c_3-b_3c_2 & b_3c_1-b_1c_3 & b_1c_2-b_2c_1\end{vmatrix}$$

$$=\left[a_2(b_1c_2-b_2c_1)-a_3(b_3c_1-b_1c_3)\right]\mathbf{i}-\left[a_1(b_1c_2-b_2c_1)-a_3(b_2c_3-b_3c_2)\right]\mathbf{j}+\left[a_1(b_3c_1-b_1c_3)-a_2(b_2c_3-b_3c_2)\right]\mathbf{k}$$

On the other hand,

$$(\mathbf{a}\cdot\mathbf{c})\,\mathbf{b}-(\mathbf{a}\cdot\mathbf{b})\,\mathbf{c}=(a_1c_1+a_2c_2+a_3c_3)(b_1\mathbf{i}+b_2\mathbf{j}+b_3\mathbf{k})-(a_1b_1+a_2b_2+a_3b_3)(c_1\mathbf{i}+c_2\mathbf{j}+c_3\mathbf{k})$$

$$=(a_1b_1c_1+a_2b_1c_2+a_3b_1c_3-a_1b_1c_1-a_2b_2c_1-a_3b_3c_1)\,\mathbf{i}+(a_1b_2c_1+a_2b_2c_2+a_3b_2c_3-a_1b_1c_2-a_2b_2c_2-a_3b_3c_2)\,\mathbf{j}+(a_1b_3c_1+a_2b_3c_2+a_3b_3c_3-a_1b_1c_3-a_2b_2c_3-a_3b_3c_3)\,\mathbf{k}$$

$$=(a_2b_1c_2+a_3b_1c_3-a_2b_2c_1-a_3b_3c_1)\,\mathbf{i}-(a_1b_1c_2+a_3b_3c_2-a_1b_2c_1-a_3b_2c_3)\,\mathbf{j}+(a_1b_3c_1+a_2b_3c_2-a_1b_1c_3-a_2b_2c_3)\,\mathbf{k}$$

The result follows when we compare the two expressions.

11.5

7. Let $A\,(2,1,4)$ and $B\,(1,3,7)$. Then $\mathbf{v}=\overrightarrow{AB}=\langle -1,2,3\rangle$ is parallel to the required line. Thus, parametric equations of the line are $x=2-t$, $y=1+2t$, $z=4+3t$ and symmetric equations are $\dfrac{x-2}{-1}=\dfrac{y-1}{2}=\dfrac{z-4}{3}$.

32. Let $A\,(2,3,-1)$, $B\,(1,-2,3)$, and $C\,(-1,2,4)$. Then $\overrightarrow{AB}=\langle -1,-5,4\rangle$ and $\overrightarrow{AC}=\langle -3,-1,5\rangle$. A normal to the required plane is $\mathbf{n}=\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ -1 & -5 & 4\\ -3 & -1 & 5\end{vmatrix}=\langle -21,-7,-14\rangle=-7\,\langle 3,1,2\rangle$, so an equation of the plane is $3\,(x-2)+1\,(y-3)+2\,(z+1)=0 \Leftrightarrow 3x+y+2z=7$.

62. A vector parallel to the line passing through $(-1,3,-1)$ and $(1,2,3)$ is $\mathbf{u}=\langle 2,-1,4\rangle$. Taking $P\,(1,4,2)$ and $Q\,(1,2,3)$ and using the result of Exercise 59, we find $\overrightarrow{QP}\times\mathbf{u}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & 2 & -1\\ 2 & -1 & 4\end{vmatrix}=7\mathbf{i}-2\mathbf{j}-4\mathbf{k}$, so

$$D=\frac{\left|\overrightarrow{QP}\times\mathbf{u}\right|}{|\mathbf{u}|}=\frac{|7\mathbf{i}-2\mathbf{j}-4\mathbf{k}|}{\sqrt{4+1+16}}=\frac{\sqrt{69}}{\sqrt{21}}=\frac{\sqrt{161}}{7}.$$

63. Let us denote the planes by $\pi_1 : ax+by+cz=d_1$ and $\pi_2 : ax+by+cz=d_2$. Choose a point $P\,(x_0,y_0,z_0)$ on π_1 and observe that a normal to π_2 is $\mathbf{n}=\langle a,b,c\rangle$. So, using the result of Exercise 59, $D=\dfrac{|ax_0+by_0+cz_0-d_2|}{\sqrt{a^2+b^2+c^2}}$. But since $P\,(x_0,y_0,z_0)$ lies on π_1, it satisfies the equation of the plane; that is, $ax_0+by_0+cz_0=d_1$. Thus, $D=\dfrac{|d_1-d_2|}{\sqrt{a^2+b^2+c^2}}$.