11.3

5. $\mathbf{a} \cdot \mathbf{b} = (2\mathbf{i} + 3\mathbf{j}) \cdot (\mathbf{i} - 2\mathbf{j}) = 2 - 6 = -4$

7. $\mathbf{a} \cdot \mathbf{b} = \langle 0, 1, -3 \rangle \cdot \langle 10, \pi, -\pi \rangle = \pi + 3\pi = 4\pi$

23. $\cos 45^\circ = \dfrac{\langle 1, c \rangle \cdot \langle 1, 2 \rangle}{\sqrt{1+c^2}\sqrt{1+4^2}} \Rightarrow \dfrac{\sqrt{2}}{2} = \dfrac{1+2c}{\sqrt{1+c^2}\sqrt{5}} \Rightarrow \sqrt{10}\sqrt{1+c^2} = 2 + 4c \Rightarrow 10\left(1+c^2\right) = 4 + 16c + 16c^2 \Rightarrow$ $6c^2 + 16c - 6 = 0 \Rightarrow (3c-1)(c+3) = 0 \Rightarrow c = -3$ or $\frac{1}{3}$. If $c = \frac{1}{3}$, then $\theta = 45^\circ$, and if $c = -3$, then $\theta = 135^\circ$, so the desired value of c is $\frac{1}{3}$.

31. If $\langle c, 2, -1 \rangle$ and $\langle 2, 3, c \rangle$ are orthogonal, then $\langle c, 2, -1 \rangle \cdot \langle 2, 3, c \rangle = 0$; that is, $2c + 6 - c = 0 \Leftrightarrow c = -6$.

32. Let $\mathbf{v} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ be a vector orthogonal to both $\mathbf{a} = \mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\mathbf{b} = -2\mathbf{i} + \mathbf{k}$. Then

$$\left.\begin{aligned} \mathbf{v} \cdot \mathbf{a} &= a + b + c = 0 \\ \mathbf{v} \cdot \mathbf{b} &= -2a \quad\;\; + c = 0 \end{aligned}\right\}$$

Solving the system, we find $a = \frac{1}{2}c$ and $b = -\frac{3}{2}c$, so $\mathbf{v} = \frac{1}{2}c\mathbf{i} - \frac{3}{2}c\mathbf{j} + c\mathbf{k} \Rightarrow$ $|\mathbf{v}| = \sqrt{\frac{1}{4}c^2 + \frac{9}{4}c^2 + c^2} = \frac{\sqrt{14}}{2}|c|$. Thus, one such unit vector is $\dfrac{\mathbf{v}}{|\mathbf{v}|} = \frac{\sqrt{14}}{14}\mathbf{i} - \frac{3\sqrt{14}}{14}\mathbf{j} + \frac{\sqrt{14}}{7}\mathbf{k}$. Note that the other unit vector is $-\dfrac{\mathbf{v}}{|\mathbf{v}|} = -\frac{\sqrt{14}}{14}\mathbf{i} + \frac{3\sqrt{14}}{14}\mathbf{j} - \frac{\sqrt{14}}{7}\mathbf{k}$.

37. $\alpha = \frac{\pi}{3}$ and $\gamma = \frac{\pi}{4}$. Since $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$, we have $\cos^2\left(\frac{\pi}{3}\right) + \cos^2\beta + \cos^2\left(\frac{\pi}{4}\right) = 1 \Rightarrow$ $\cos^2\beta = 1 - \left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{1}{4}$. Thus, $\cos\beta = \pm\frac{1}{2}$, and so $\beta = \frac{\pi}{3}$ or $\frac{2\pi}{3}$.

41. $\mathbf{a} = 2\mathbf{i} + \mathbf{j} + 4\mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} + \mathbf{k}$.

a. $\operatorname{proj}_{\mathbf{a}} \mathbf{b} = \left(\dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|^2}\right)\mathbf{a} = \left[\dfrac{(2\mathbf{i} + \mathbf{j} + 4\mathbf{k}) \cdot (3\mathbf{i} + \mathbf{k})}{4 + 1 + 16}\right](2\mathbf{i} + \mathbf{j} + 4\mathbf{k}) = \frac{6+4}{21}(2\mathbf{i} + \mathbf{j} + 4\mathbf{k}) = \frac{20}{21}\mathbf{i} + \frac{10}{21}\mathbf{j} + \frac{40}{21}\mathbf{k}$

b. $\operatorname{proj}_{\mathbf{b}} \mathbf{a} = \left(\dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|^2}\right)\mathbf{b} = \left[\dfrac{(2\mathbf{i} + \mathbf{j} + 4\mathbf{k}) \cdot (3\mathbf{i} + \mathbf{k})}{9 + 1}\right](3\mathbf{i} + \mathbf{k}) = \frac{6+4}{10}(3\mathbf{i} + \mathbf{k}) = 3\mathbf{i} + \mathbf{k}$

43. $\mathbf{a} = \langle -3, 4, -2 \rangle$ and $\mathbf{b} = \langle 0, 1, 0 \rangle$.

a. $\operatorname{proj}_{\mathbf{a}} \mathbf{b} = \left(\dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|^2}\right)\mathbf{a} = \left(\dfrac{\langle -3, 4, -2 \rangle \cdot \langle 0, 1, 0 \rangle}{9 + 16 + 4}\right)\langle -3, 4, -2 \rangle = \frac{4}{29}\langle -3, 4, -2 \rangle = \left\langle -\frac{12}{29}, \frac{16}{29}, -\frac{8}{29} \right\rangle$

b. $\operatorname{proj}_{\mathbf{b}} \mathbf{a} = \left(\dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|^2}\right)\mathbf{b} = \left(\dfrac{\langle -3, 4, -2 \rangle \cdot \langle 0, 1, 0 \rangle}{1}\right)\langle 0, 1, 0 \rangle = 4\langle 0, 1, 0 \rangle = \langle 0, 4, 0 \rangle$

11.4

3. $\mathbf{a}\times\mathbf{b}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\1&1&0\\0&2&3\end{vmatrix}=3\mathbf{i}-3\mathbf{j}+2\mathbf{k}$

5. $\mathbf{a}\times\mathbf{b}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\1&-2&1\\3&1&-2\end{vmatrix}=3\mathbf{i}+5\mathbf{j}+7\mathbf{k}$

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15. $\mathbf{a}\times\mathbf{b}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\-3&1&-2\\1&1&1\end{vmatrix}=3\mathbf{i}+\mathbf{j}-4\mathbf{k}$, so the required unit vectors are $\pm\left(\dfrac{3\mathbf{i}+\mathbf{j}-4\mathbf{k}}{\sqrt{9+1+16}}\right)$; that is, $\pm\frac{\sqrt{26}}{26}\,(3\mathbf{i}+\mathbf{j}-4\mathbf{k})$.

19. For $P\,(1,-1,2)$, $Q\,(2,3,1)$, and $R\,(-2,3,4)$, $\overrightarrow{PQ}=\mathbf{i}+4\mathbf{j}-\mathbf{k}$ and $\overrightarrow{PR}=-3\mathbf{i}+4\mathbf{j}+2\mathbf{k}$,

so $\overrightarrow{PQ}\times\overrightarrow{PR}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\1&4&-1\\-3&4&2\end{vmatrix}=12\mathbf{i}+\mathbf{j}+16\mathbf{k}$, so the area of $\triangle PQR$ is

27. $\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})=\begin{vmatrix}1&1&0\\0&1&-2\\1&2&3\end{vmatrix}=5\Rightarrow V=|\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})|=|5|=5$

29. For $P\,(0,0,0)$, $Q\,(3,-2,1)$, $R\,(1,2,2)$, and $S\,(1,1,4)$, $\overrightarrow{PQ}=\langle 3,-2,1\rangle$, $\overrightarrow{PR}=\langle 1,2,2\rangle$, and $\overrightarrow{PS}=\langle 1,1,4\rangle$, so

$\overrightarrow{PQ}\cdot\left(\overrightarrow{PR}\times\overrightarrow{PS}\right)=\begin{vmatrix}3&-2&1\\1&2&2\\1&1&4\end{vmatrix}=21$. Thus, $V=\left|\overrightarrow{PQ}\cdot\left(\overrightarrow{PR}\times\overrightarrow{PS}\right)\right|=|21|=21$.

54. Let $\mathbf{v}=a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$ be a vector in the desired direction. Since $\mathbf{v}$ lies in the plane determined by $\mathbf{a}=\mathbf{i}+3\mathbf{j}+2\mathbf{k}$ and

$\mathbf{b}=-\mathbf{i}+2\mathbf{j}+4\mathbf{k}$, $\mathbf{v}$, $\mathbf{a}$, and $\mathbf{b}$ are coplanar, so $\mathbf{v}\cdot(\mathbf{a}\times\mathbf{b})=0\Rightarrow\mathbf{v}\cdot(\mathbf{a}\times\mathbf{b})=\begin{vmatrix}a&b&c\\1&3&2\\-1&2&4\end{vmatrix}=8a-6b+5c=0$ (1). Also,

$\mathbf{v}$ is perpendicular to $\mathbf{c}=3\mathbf{i}-\mathbf{j}+2\mathbf{k}$, so $\mathbf{v}\cdot\mathbf{c}=0\Rightarrow\mathbf{v}\cdot\mathbf{c}=(a\mathbf{i}+b\mathbf{j}+c\mathbf{k})\cdot(3\mathbf{i}-\mathbf{j}+2\mathbf{k})=3a-b+2c=0$ (2). Solving

the system $\left.\begin{aligned}8a-6b+5c&=0\\3a-\ \ b+2c&=0\end{aligned}\right\}$ for a and b in terms of c gives $a=-\frac{7}{10}c$ and $b=-\frac{1}{10}c$, so $\mathbf{v}=-\frac{7}{10}c\mathbf{i}-\frac{1}{10}c\mathbf{j}+c\mathbf{k}$.

We may take $c=-10$. Then the vector $\mathbf{w}=7\mathbf{i}+\mathbf{j}-10\mathbf{k}$ also satisfies the requirements, and the desired unit vector is

$\mathbf{u}=\dfrac{\mathbf{w}}{|\mathbf{w}|}=\frac{1}{5\sqrt{6}}\mathbf{w}=\frac{\sqrt{6}}{30}\,(7\mathbf{i}+\mathbf{j}-10\mathbf{k})=\frac{7\sqrt{6}}{30}\mathbf{i}+\frac{\sqrt{6}}{30}\mathbf{j}-\frac{\sqrt{6}}{3}\mathbf{k}$.

55. If $\mathbf{a}=\langle -1,a,3\rangle$ and $\mathbf{b}=\langle 2,3,b\rangle$ are parallel, then $\mathbf{a}\times\mathbf{b}=\mathbf{0}\Leftrightarrow$

$\mathbf{a}\times\mathbf{b}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\-1&a&3\\2&3&b\end{vmatrix}=(ab-9)\,\mathbf{i}-(-b-6)\,\mathbf{j}+(-3-2a)\,\mathbf{k}=0\mathbf{i}+0\mathbf{j}+0\mathbf{k}\Leftrightarrow ab-9=0$, $b+6=0$, and

$-3-2a=0$. Solving, we find $a=-\frac{3}{2}$ and $b=-6$.

11.5

3. Parametric equations: $x = 1 + 2t$, $y = 3 + 4t$, $z = 2 + 5t$. Symmetric equations: $\dfrac{x-1}{2} = \dfrac{y-3}{4} = \dfrac{z-2}{5}$.

7. Let $A(2, 1, 4)$ and $B(1, 3, 7)$. Then $\mathbf{v} = \overrightarrow{AB} = \langle -1, 2, 3\rangle$ is parallel to the required line. Thus, parametric equations of the line are $x = 2 - t$, $y = 1 + 2t$, $z = 4 + 3t$ and symmetric equations are $\dfrac{x-2}{-1} = \dfrac{y-1}{2} = \dfrac{z-4}{3}$.

14. Suppose L_1 and L_2 intersect at the point $P_0(x_0, y_0, z_0)$. Then there exist numbers t_1 and t_2 such that

$$\left.\begin{aligned} 4 + t_1 &= 6 + 2t_2 \\ 5 + t_1 &= 11 + 4t_2 \\ -1 + 2t_1 &= -3 + t_2 \end{aligned}\right\}$$

Solving the first two equations simultaneously, we find $t_1 = -2$ and $t_2 = -2$. Substituting these values into the third equation gives $-5 = -5$, so the solution to the system is $t_1 = t_2 = -2$. Therefore the point of intersection is $(2, 3, -5)$. The required line is parallel to the vector $\mathbf{v} = 4\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$ and so parametric equations of the required line are $x = 2 + 4t$, $y = 3 + 5t$, $z = -5 + 2t$.

27. A normal to the given plane is $\mathbf{n} = \langle 2, 3, -1\rangle$. Because the required plane is parallel to the given plane, $\mathbf{n}$ is also normal to the required plane, so an equation is $2(x-3) + 3(y-6) - 1(z+2) = 0 \Leftrightarrow 2x + 3y - z = 26$.

31. Let $A(1, 0, -2)$, $B(1, 3, 2)$, and $C(2, 3, 0)$. Then $\overrightarrow{AB} = \langle 0, 3, 4\rangle$ and $\overrightarrow{AC} = \langle 1, 3, 2\rangle$. A normal to the required plane is

$$\mathbf{n} = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 3 & 4 \\ 1 & 3 & 2 \end{vmatrix} = \langle -6, 4, -3\rangle,$$ so an equation of the plane is $-6(x-1) + 4(y-0) - 3(z+2) = 0 \Leftrightarrow$ $6x - 4y + 3z = 0$.

33. Let $P(1, 3, 2)$. Setting $t = 0$ gives the point $Q(1, -1, 3)$ on the line. Also, a vector in the same direction as the line is $\mathbf{v} = \langle 1, -2, 2\rangle$, so a vector normal to the required plane is $\mathbf{n} = \overrightarrow{PQ} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -4 & 1 \\ 1 & -2 & 2 \end{vmatrix} = -6\mathbf{i} + \mathbf{j} + 4\mathbf{k}$, and an equation of the plane is $-6(x-1) + 1(y-3) + 4(z-2) = 0 \Leftrightarrow 6x - y - 4z = -5$.

37. Let $P(2, 1, 1)$ and $Q(-1, 3, 2)$, so $\overrightarrow{PQ} = \langle -3, 2, 1\rangle$. If $\mathbf{n} = \langle 2, 3, -4\rangle$ is normal to the given plane, then a normal to the required plane is $\overrightarrow{PQ} \times \mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & 2 & 1 \\ 2 & 3 & -4 \end{vmatrix} = -11\mathbf{i} - 10\mathbf{j} - 13\mathbf{k} = -(11\mathbf{i} + 10\mathbf{j} + 13\mathbf{k})$. Thus, an equation of the required plane is $11(x-2) + 10(y-1) + 13(z-1) = 0 \Leftrightarrow 11x + 10y + 13z = 45$.

43. A normal to the plane $x + y + 2z = 6$ is $\mathbf{n} = \langle 1, 1, 2\rangle$ and a vector parallel to the line $L: x = 1 + t$, $y = 2 + t$, $z = -1 + t$ is $\mathbf{v} = \langle 1, 1, 1\rangle$, so the angle between the normal to the plane and the line is $\theta = \cos^{-1}\left(\dfrac{|\mathbf{n}\cdot\mathbf{v}|}{|\mathbf{n}|\,|\mathbf{v}|}\right) = \cos^{-1}\left(\dfrac{|\langle 1,1,2\rangle \cdot \langle 1,1,1\rangle|}{\sqrt{1+1+4}\sqrt{1+1+1}}\right) = \cos^{-1}\dfrac{4}{\sqrt{6}\sqrt{3}} \approx 19.5^\circ$. Therefore, the required angle is about $90^\circ - 19.5^\circ = 70.5^\circ$.

47. A normal to the plane $2x + 4y - 3z = 4$ is $\mathbf{n} = \langle 2, 4, -3\rangle$. Since the line is perpendicular to the plane, its direction is given by $\mathbf{v} = \mathbf{n}$, so parametric equations of the line are $x = 2 + 2t$, $y = 3 + 4t$, $z = -1 - 3t$.

50. The line of intersection of the two planes is defined by the system $\left.\begin{array}{r} x - \;\; y + 2z = 1 \\ 2x + 3y - \;\; z = 2 \end{array}\right\}$ To find two distinct points on the line, we take $z = 0$ and $z = 1$ successively to obtain $\left.\begin{array}{r} x - \;\; y = 1 \\ 2x + 3y = 2 \end{array}\right\}$ and $\left.\begin{array}{r} x - \;\; y = -1 \\ 2x + 3y = \;\;\; 3 \end{array}\right\}$ Solving each system, we find the points to be $P(1, 0, 0)$ and $Q(0, 1, 1)$. Letting $R(3, 4, 1)$ denote the given point, we see that a normal to the required plane is $\overrightarrow{RP} \times \overrightarrow{RQ} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & -4 & -1 \\ -3 & -3 & 0 \end{vmatrix} = -3\mathbf{i} + 3\mathbf{j} - 6\mathbf{k} = -3(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$. Taking $\mathbf{n} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $R(3, 4, 1)$, we find an equation of the required plane to be $(x - 3) - (y - 4) + 2(z - 1) = 0 \Leftrightarrow x - y + 2z = 1$.

55. To find the distance between $P(3, 1, 2)$ and the plane $2x - 3y + 4z = 7 \Leftrightarrow 2x - 3y + 4z - 7 = 0$, we write $\mathbf{n} = \langle a, b, c\rangle = \langle 2, -3, 4\rangle \Rightarrow D = \dfrac{|2(3) - 3(1) + 4(2) - 7|}{\sqrt{2^2 + 3^2 + 4^2}} = \dfrac{4\sqrt{29}}{29}$.