5.3 Volumes Using Cylindrical Shells

3. $V = 2\pi \int_0^1 x\left(x - x^2\right)dx = 2\pi \int_0^1 \left(x^2 - x^3\right)dx = 2\pi\left(\frac{1}{3}x^3 - \frac{1}{4}x^4\right)\Big|_0^1 = \frac{\pi}{6}$

5. $V = 2\pi \int_0^4 y\left(y^{1/2} - \frac{1}{8}y^2\right)dy = 2\pi \int_0^4 \left(y^{3/2} - \frac{1}{8}y^3\right)dy = 2\pi\left(\frac{2}{5}y^{5/2} - \frac{1}{32}y^4\right)\Big|_0^4 = \frac{48\pi}{5}$

6. $V = 2\pi \int_0^2 (3 - x)\left[\left(\frac{1}{2}x^2 + 2\right) - x^2\right]dx = 2\pi \int_0^2 \left(\frac{1}{2}x^3 - \frac{3}{2}x^2 - 2x + 6\right)dx = 2\pi\left(\frac{1}{8}x^4 - \frac{1}{2}x^3 - x^2 + 6x\right)\Big|_0^2 = 12\pi$

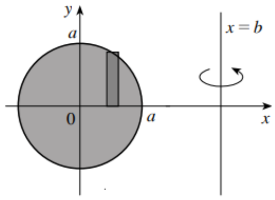
43. Using symmetry, we obtain

$$V = 2 \cdot 2\pi \int_{-a}^{a} (b - x)\, y\, dx = 4\pi \int_{-a}^{a} (b - x)\left(a^2 - x^2\right)^{1/2} dx$$

$$= 4\pi b \int_{-a}^{a} \left(a^2 - x^2\right)^{1/2} dx - 4\pi \int_{-a}^{a} x\left(a^2 - x^2\right)^{1/2} dx$$

Interpreting the integral geometrically, we find the first integral to be equal to $4\pi b\left(\frac{1}{2}\pi a^2\right)$. The second integral is 0 because the integrand is odd.

Thus, $V = 2\pi^2 a^2 b$.



6.1 The Natural Logarithmic Function

33. $y = x(\ln x)^2 \Rightarrow y' = x\frac{d}{dx}(\ln x)^2 + (\ln x)^2 \cdot \frac{d}{dx}(x) = x \cdot 2\ln x \cdot \frac{1}{x} + (\ln x)^2 = (\ln x)^2 + 2\ln x$

34. $f(x) = \ln\left(x + \sqrt{x^2 - 1}\right) \Rightarrow$

$$f'(x) = \frac{1 + \frac{1}{2}\left(x^2 - 1\right)^{-1/2}(2x)}{x + \left(x^2 - 1\right)^{1/2}} = \frac{1 + x\left(x^2 - 1\right)^{-1/2}}{x + \left(x^2 - 1\right)^{1/2}} = \frac{1 + \dfrac{x}{\sqrt{x^2 - 1}}}{x + \sqrt{x^2 - 1}} = \frac{\dfrac{\sqrt{x^2 - 1} + x}{\sqrt{x^2 - 1}}}{x + \sqrt{x^2 - 1}} = \frac{1}{\sqrt{x^2 - 1}}$$

65. $y = (2x + 1)^2\left(3x^2 - 4\right)^3 \Rightarrow \ln y = 2\ln(2x + 1) + 3\ln\left(3x^2 - 4\right) \Rightarrow \frac{y'}{y} = \frac{2 \cdot 2}{2x + 1} + \frac{3 \cdot 6x}{3x^2 - 4} = \frac{2\left(24x^2 + 9x - 8\right)}{(2x + 1)\left(3x^2 - 4\right)}$

$\Rightarrow y' = 2(2x + 1)\left(3x^2 - 4\right)^2\left(24x^2 + 9x - 8\right)$

79. Let $u = 1 + \sin x$. Then $du = \cos x\, dx$, so $\int \frac{\cos x}{1 + \sin x}dx = \int \frac{du}{u} = \ln|u| + C = \ln|1 + \sin x| + C$.