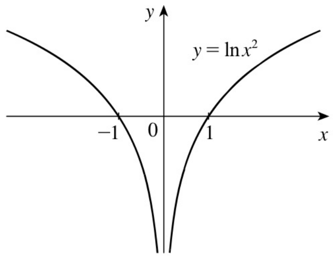
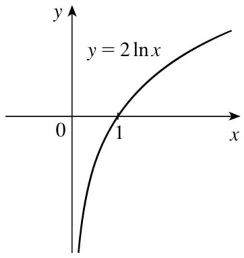
6.1 Concept Questions

1. See page 518. Its domain is $(0, \infty)$ and its range is $(-\infty, \infty)$.

2. See page 519.

3. No. The function f has domain $(-\infty, 0) \cup (0, \infty)$, whereas the domain of g is $(0, \infty)$.





4. $f(-x) = \ln\left(-x + \sqrt{1 + (-x)^2}\right) = \ln\left(\sqrt{1+x^2} - x\right) = \ln\left(\dfrac{\sqrt{1+x^2} - x}{1} \cdot \dfrac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}+x}\right)$

$= \ln \dfrac{1}{\sqrt{1+x^2}+x} = \ln\left(\sqrt{1+x^2}+x\right)^{-1} = -\ln\left(x + \sqrt{1+x^2}\right) = -f(x),$

so f is an odd function.

28. $g(x) = \ln\left(x^2+4\right)^2 = 2\ln\left(x^2+4\right) \Rightarrow g'(x) = 2\dfrac{d}{dx}\ln\left(x^2+4\right) = \dfrac{2(2x)}{x^2+4} = \dfrac{4x}{x^2+4}$

32. $g(t) = t\ln 2t \Rightarrow g'(t) = t \cdot \dfrac{d}{dt}\ln 2t + (\ln 2t)\cdot\dfrac{d}{dt}(t) = t \cdot \dfrac{1}{t} + \ln 2t = 1 + \ln 2t$

34. $f(x) = \ln\left(x + \sqrt{x^2-1}\right) \Rightarrow$

$$f'(x) = \frac{1 + \frac{1}{2}\left(x^2-1\right)^{-1/2}(2x)}{x + \left(x^2-1\right)^{1/2}} = \frac{1 + x\left(x^2-1\right)^{-1/2}}{x + \left(x^2-1\right)^{1/2}} = \frac{1 + \dfrac{x}{\sqrt{x^2-1}}}{x + \sqrt{x^2-1}} = \frac{\dfrac{\sqrt{x^2-1}+x}{\sqrt{x^2-1}}}{x + \sqrt{x^2-1}} = \frac{1}{\sqrt{x^2-1}}$$

44. $g'(\theta) = \dfrac{d}{d\theta}\ln|\tan 3\theta| = \dfrac{3\sec^2 3\theta}{\tan 3\theta} = \dfrac{3\cos 3\theta}{\cos^2 3\theta \cdot \sin 3\theta} = \dfrac{3}{\cos 3\theta \sin 3\theta}$

50. $\ln xy - y^2 = 5 \Rightarrow \ln x + \ln y - y^2 = 5$. Differentiating, we obtain $\dfrac{1}{x} + \dfrac{y'}{y} - 2yy' = 0 \Rightarrow y'\left(\dfrac{1}{y} - 2y\right) = -\dfrac{1}{x} \Rightarrow$

$y' = \dfrac{y}{x\left(2y^2-1\right)}$.

52. $\ln(x+y) - \cos y - x^2 = 0 \Rightarrow \dfrac{1+y'}{x+y} + (\sin y)\,y' - 2x = 0 \Rightarrow \dfrac{1}{x+y} - 2x + \left(\dfrac{1}{x+y} + \sin y\right)y' = 0 \Rightarrow$

$\dfrac{1 - 2x(x+y)}{x+y} + \dfrac{1 + (x+y)\sin y}{x+y}y' = 0 \Rightarrow y' = \dfrac{2x(x+y) - 1}{1 + (x+y)\sin y}$

56. $y - \ln\left(x^2+y^2\right) = 0 \Rightarrow y' - \dfrac{2x + 2yy'}{x^2+y^2} = 0$. Substituting $x = 1$ and $y = 0$ into the equation gives $y' - \dfrac{2+0}{1+0} = 0$ or

$y' = 2$, the slope of the required tangent line. An equation is $y - 0 = 2(x-1)$ or $y = 2x - 2$.

66. $y = \dfrac{x^2\sqrt{2x-4}}{(x+1)^2} \Rightarrow \ln y = 2\ln x + \frac{1}{2}\ln(2x-4) - 2\ln(x+1) \Rightarrow \dfrac{y'}{y} = \dfrac{2}{x} + \dfrac{1}{2x-4} - \dfrac{2}{x+1} = \dfrac{x^2+5x-8}{2x(x-2)(x+1)} \Rightarrow$

$y' = \dfrac{x\left(x^2+5x-8\right)}{\sqrt{2x-4}\,(x+1)^3}$

68. $y = \dfrac{\sin^2 x}{x^2\sqrt{1+\tan x}} \Rightarrow \ln y = 2\ln\sin x - 2\ln x - \frac{1}{2}\ln(1+\tan x) \Rightarrow \dfrac{y'}{y} = 2\dfrac{\cos x}{\sin x} - \dfrac{2}{x} - \dfrac{1}{2}\cdot\dfrac{\sec^2 x}{1+\tan x} \Rightarrow$

$y' = \left[2\cot x - \dfrac{2}{x} - \dfrac{\sec^2 x}{2(1+\tan x)}\right]\dfrac{\sin^2 x}{x^2\sqrt{1+\tan x}}$

69. Taking logarithms of both sides gives $\ln y = \ln x^x = x \ln x$, so $\frac{y'}{y} = x\left(\frac{1}{x}\right) + \ln x = 1 + \ln x \Rightarrow y' = (\ln x + 1)\, x^x$.

Therefore, $y'' = (\ln x + 1)\frac{d}{dx}x^x + x^x\frac{d}{dx}(\ln x + 1) = (\ln x + 1)(\ln x + 1)\,x^x + x^x\left(\frac{1}{x}\right) = \left[x(\ln x + 1)^2 + 1\right]x^{x-1}$.

72. Let $u = 2x + 3$. Then $du = 2\,dx$, so $\displaystyle\int \frac{1}{2x+3}\,dx = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|2x+3| + C$.

74. $\displaystyle\int_1^3 \frac{x^2 - x + 3}{x}\,dx = \int_1^3 \left(x - 1 + \frac{3}{x}\right)dx = \left(\frac{1}{2}x^2 - x + 3\ln|x|\right)\Big|_1^3 = 2 + 3\ln 3$

76. Let $u = \ln x$. Then $du = \dfrac{dx}{x}$, $x = 1 \Rightarrow u = 0$, and $x = 3 \Rightarrow u = \ln 3$. Thus,
$\displaystyle\int_1^3 \frac{\ln x}{x}\,dx = \int_0^{\ln 3} u\,du = \frac{1}{2}u^2\Big|_0^{\ln 3} = \frac{1}{2}(\ln 3)^2$.

78. Let $u = 1 + \ln x$. Then $du = \dfrac{dx}{x}$, so $\displaystyle\int \frac{\sqrt{1+\ln x}}{x}\,dx = \int \sqrt{u}\,du = \frac{2}{3}u^{3/2} + C = \frac{2}{3}(1 + \ln x)^{3/2} + C$.

80. Let $u = 4 - \tan 3x$. Then $du = -3\sec^2 3x\,dx$, so $\displaystyle\int \frac{\sec^2 3x}{4 - \tan 3x}\,dx = -\frac{1}{3}\int \frac{du}{u} = -\frac{1}{3}\ln|u| + C = -\frac{1}{3}\ln|4 - \tan 3x| + C$.

82. Let $u = 1 + \sin^2 x$. Then $du = 2\sin x\cos x\,dx = \sin 2x\,dx$, so
$\displaystyle\int \frac{\sin 2x}{1 + \sin^2 x}\,dx = \int \frac{du}{u} = \ln|u| + C = \ln\left(1 + \sin^2 x\right) + C$.

84. Let $u = \ln x$. Then $du = \dfrac{dx}{x}$, so $\displaystyle I = \int \frac{\ln x\sqrt{1+\ln x}}{x}\,dx = \int u(1+u)^{1/2}\,du$. Now let $v = 1 + u$. Then $dv = du$ and
$I = \int (v-1)\,v^{1/2}\,dv = \int \left(v^{3/2} - v^{1/2}\right)dv = \frac{2}{5}v^{5/2} - \frac{2}{3}v^{3/2} + C = \frac{2}{15}v^{3/2}(3v - 5) + C$
$= \frac{2}{15}(1+u)^{3/2}(3u - 2) + C = \frac{2}{15}(\ln x + 1)^{3/2}(3\ln x - 2) + C$

91. $\frac{dy}{dx} = \frac{d}{dx}\int_x^{x^2}\ln t\,dt = \frac{d}{dx}\left[\int_x^c \ln t\,dt + \int_c^{x^2}\ln t\,dt\right] = \frac{d}{dx}\left[-\int_c^x \ln t\,dt + \int_c^{x^2}\ln t\,dt\right] = -\ln x + \left(\ln x^2\right)\frac{d}{dx}\left(x^2\right)$
$= -\ln x + 2x\ln x^2 = (4x - 1)\ln x$

92. a. $\displaystyle y = \int_{2/x}^{x^2}\frac{dt}{t} = \ln|t|\,\Big|_{2/x}^{x^2} = \ln x^2 - \ln\frac{2}{x} = 2\ln x - \ln 2 + \ln x = 3\ln x - \ln 2$, so $\dfrac{dy}{dx} = \dfrac{d}{dx}(3\ln x - \ln 2) = \dfrac{3}{x}$.

b. $\displaystyle\frac{dy}{dx} = \frac{d}{dx}\int_{2/x}^{x^2}\frac{dt}{t} = \frac{d}{dx}\left[\int_{2/x}^{c}\frac{dt}{t} + \int_c^{x^2}\frac{dt}{t}\right] = \frac{d}{dx}\left[-\int_c^{2/x}\frac{dt}{t} + \int_c^{x^2}\frac{dt}{t}\right] = -\frac{1}{2/x}\frac{d}{dx}\left(\frac{2}{x}\right) + \frac{1}{x^2}\frac{d}{dx}\left(x^2\right)$
$\displaystyle = \left(-\frac{x}{2}\right)\left(-\frac{2}{x^2}\right) + \frac{1}{x^2}(2x) = \frac{1}{x} + \frac{2}{x} = \frac{3}{x}$

109. False. Take $a = 2$ and $b = 1$. Then $\ln a - \ln b = \ln 2 - \ln 1 = \ln 2$, but $\ln(a - b) = \ln(2 - 1) = \ln 1 = 0$, so $\ln a - \ln b \neq \ln(a - b)$.

110. False. Take $x = e$. Then $(\ln x)^3 = (\ln e)^3 = 1^3 = 1$, but $3\ln x = 3\ln e = 3 \cdot 1 = 3$.

111. True. If $x \neq 0$, then $f(x) = \ln|x| = \begin{cases} \ln x & \text{if } x > 0 \\ \ln(-x) & \text{if } x < 0 \end{cases}$

112. True. $f(x) = \ln x$ is increasing and so $0 < a < b \Rightarrow f(a) < f(b)$.

113. True. $g(x) = \ln x$ is continuous on $(1, \infty)$ and $g(x) \neq 0$ on $(1, \infty)$, so $f(x) = \dfrac{1}{g(x)} = \dfrac{1}{\ln x}$ is continuous on $(1, \infty)$.

114. False. $f(x) = \ln 5$ is a constant function, so $f'(x) = \frac{d}{dx}(\ln 5) = 0$.

115. False. The integrand is not defined at $x = 2$, so neither integral is defined.

116. False. The integrand $f(x) = 1/x$ is not defined at $x = 0$, which is in the interval of integration $[-2, 2]$.

6.2 Concept Questions

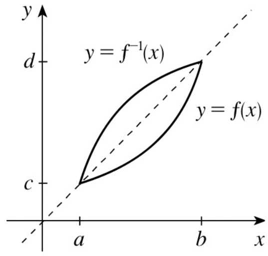
1. a. See page 535. The function $f(x) = x^3$ is one-to-one on $(-\infty, \infty)$.

b. See page 535.

2. a. $f\left(f^{-1}(x)\right) = x$ for every x in $[c, d]$ and

$f^{-1}(f(x)) = x$ for every x in $[a, b]$.

b.



3. a. See page 533.

b. See pages 533 and 534.

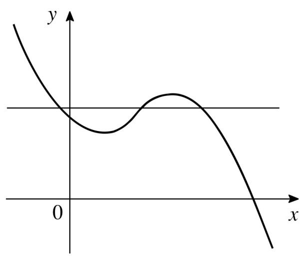
4. See page 537.

2. $f(g(x)) = f\left(\frac{1}{x}\right) = \frac{1}{1/x} = x$ and $g(f(x)) = g\left(\frac{1}{x}\right) = \frac{1}{1/x} = x.$

4. $f(g(x)) = f\left(-\sqrt{x-1}\right) = \left(-\sqrt{x-1}\right)^2 + 1 = (x-1) + 1 = x$ and
$g(f(x)) = g\left(x^2+1\right) = -\sqrt{\left(x^2+1\right)-1} = -\sqrt{x^2} = -(-x) = x$ (since $x \le 0$).

6. $f(g(x)) = f\left(\frac{x-1}{x+1}\right) = \dfrac{1+\frac{x-1}{x+1}}{1-\frac{x-1}{x+1}} = \dfrac{\frac{2x}{x+1}}{\frac{2}{x+1}} = x$ and $g(f(x)) = g\left(\frac{1+x}{1-x}\right) = \dfrac{\frac{1+x}{1-x}-1}{\frac{1+x}{1-x}+1} = \dfrac{\frac{2x}{1-x}}{\frac{2}{1-x}} = x.$

8. f is not one-to-one. The horizontal line shown cuts the graph of f at three points.



10. f is one-to-one. There is no horizontal line that cuts the graph of f at more than one point.

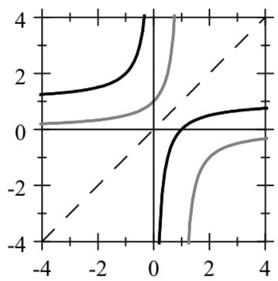
22. By inspection $f(0) = 2$, so $f^{-1}(2) = 0$.

24. By inspection $f(0) = 2$, so $f^{-1}(2) = 0$.

26. By inspection $f(1) = 1$, so $f^{-1}(1) = 1$.

36. Put $y = 1 - \frac{1}{x}$. Then $\frac{1}{x} = 1 - y \Leftrightarrow x = \frac{1}{1-y}$, so

$f^{-1}(x) = \frac{1}{1-x}$.

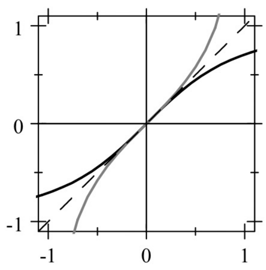


38. Put $y = \frac{x}{\sqrt{x^2+1}}$. Then $y^2 = \frac{x^2}{x^2+1} \Leftrightarrow$

$y^2\left(x^2+1\right) = x^2 \Leftrightarrow y^2x^2 + y^2 = x^2 \Leftrightarrow$

$x^2\left(1-y^2\right) = y^2 \Rightarrow x = \frac{y}{\sqrt{1-y^2}}$, so

$f^{-1}(x) = \frac{x}{\sqrt{1-x^2}}$, $-\frac{\sqrt{2}}{2} \le x \le \frac{\sqrt{2}}{2}$.



61. True. This follows from the definition of the inverse of a function.

62. False. Take $f(x) = x^{1/3}$ and $a = 8$. Then $f^{-1}(x) = x^3$ and $\left(f^{-1}\right)'(x) = 3x^2$, so $\left(f^{-1}\right)'(8) = 3\left(8^2\right) = 192$, but $f'(8) = \frac{1}{3}(8)^{-2/3} = \frac{1}{12}$, so $\frac{1}{f'(8)} = 12 \neq \left(f^{-1}\right)'(8)$.

63. True. $f(x) = 1/x^2$ is one-to-one on $(-\infty, 0)$ as well as on $(0, \infty)$, so if (a, b) is an interval not containing 0, then f is one-to-one and has an inverse.

64. True. $F'(x) = \frac{d}{dx}\int_0^x \sqrt[3]{1+t^2}\,dt = \sqrt[3]{1+x^2}$ is strictly increasing on $(0, \infty)$ and hence one-to-one (it passes the horizontal line test), so F has an inverse on $(0, \infty)$.

65. True. See Theorem 2.

66. False. Consider $f(x) = \begin{cases} 1/x & \text{if } x < 0 \\ \sqrt{x} & \text{if } x \geq 0 \end{cases}$ Then f is not monotonic, but $f^{-1}(x) = \begin{cases} 1/x & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$

67. True. $f'(x) = (2n+1)\,a_{2n+1}x^{2n} + (2n-1)\,a_{2n-1}x^{2n-2} + (2n-3)\,a_2x^{2n-4} + \cdots + a_1 > 0 \Rightarrow f$ is monotonically increasing $\Rightarrow f'$ exists.

68. False. Suppose f has domain $\{1\}$ and $f(1) = 1$.