聯微作業解答 (4.5, 5.1)

4.5

- Find the derivative of the function.

10. $h(x) = \int_0^{x^2} \sin t^2 dt$

Sol. $h'(x) = \sin(x^2)^2(2x) = 2x \sin x^4$

11. $F(x) = \int_1^{\cos x} \dfrac{t^2}{t+1} dt$

Sol. $F'(x) = \dfrac{\cos^2 x}{\cos x + 1}(-\sin x) = \dfrac{-\sin x \cos^2 x}{\cos x + 1}$

- Evaluate the integral.

22. $\int_1^0 (t^{\frac{1}{2}} - t^{\frac{5}{2}}) dt$

Sol. $\int_1^0 (t^{\frac{1}{2}} - t^{\frac{5}{2}}) dt = \frac{2}{3} t^{\frac{3}{2}} - \frac{2}{7} t^{\frac{7}{2}} |_1^0 = 0 - (\frac{2}{3} - \frac{2}{7}) = \frac{-8}{21},$

88. Find $\dfrac{dx}{dy}$ if $\int_0^x \sqrt{3 + 2\cos t} dt + \int_0^y \sin t dt = 0.$

Sol. $\dfrac{d}{dy}(\int_0^x \sqrt{3 + 2\cos t} dt + \int_0^y \sin t dt) = \dfrac{d}{dy}(0)$

$\Rightarrow \sqrt{3 + 2\cos x}\dfrac{dx}{dy} + \sin y = 0 \Rightarrow \dfrac{dx}{dy} = \dfrac{-\sin y}{\sqrt{3 + 2\cos x}}$

89. Find the x-coordinates of the relative extrema of the function
$F(x) = \int_0^x \frac{\sin t}{t} dt,\ x > 0.$

Sol. $F'(x) = \frac{\sin x}{x}$, $F'(x) = 0 \Rightarrow x = n\pi$ for $n = 1, 2, \cdots$

$F''(x) = \frac{x\cos x - \sin x}{x^2}$,

$F''(2n\pi) = \frac{1}{2n\pi} > 0$, and $F''((2n-1)\pi) = \frac{-1}{(2n-1)\pi} < 0$

By second Derivative Test, F has relative maximum at $x = (2n-1)\pi$ and relative minimum at $x = 2n\pi$, $n = 1, 2, \cdots$.

• Skrtch the region bounded by the graphs of the given equations and find the area of that region.

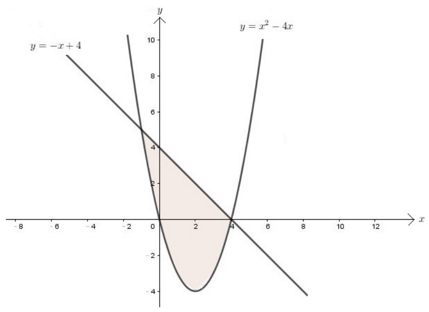
12. $y = x^2 - 4x$, $y = -x + 4$

Sol. Let $x^2 - 4x = -x + 4 \Rightarrow x^2 - 3x - 4 = 0 \Rightarrow (x-4)(x+1) = 0$

$\Rightarrow x = 4$ or $x = -1$. So $(4, 0)$ and $(-1, 5)$ are the points of intersection of given equations. Thus

$A = \int_{-1}^{4}((-x+4) - (x^2 - 4x))dx = \int_{-1}^{4}(-x^2 + 3x + 4)dx$

$=(\frac{-1}{3}x^3 + \frac{3}{2}x^2 + 4x)|_{-1}^{4} = \frac{125}{6}$



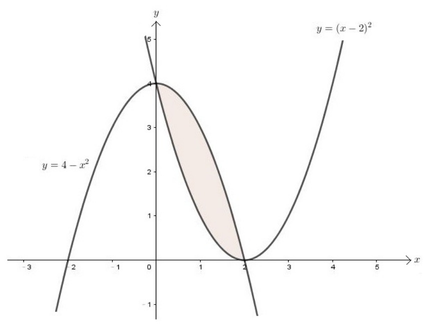
14. $y = (x-2)^2$, $y = 4 - x^2$

Sol. $(x-2)^2 = 4 - x^2 \Rightarrow 2x^2 - 4x = 0 \Rightarrow 2x(x-2) = 0$

$\Rightarrow x = 0$ or $x = 2$. So $(0, 4)$, and $(2, 0)$ are the points of intersection of given equations. Thus

$A = \int_{0}^{2}((4 - x^2) - (x-2)^2)dx = \int_{0}^{2}(-2x^2 + 4x)dx$

$= (-\frac{2}{3}x^3 + 2x^2)|_{0}^{2} = \frac{8}{3}$



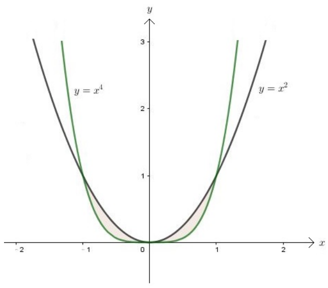
16. $y = x^2$, $y = x^4$

Sol. $x^2 = x^4 \Rightarrow x^4 - x^2 = 0 \Rightarrow x^2(x+1)(x-1) = 0$

$\Rightarrow x = 0$, $x = 1$ or $x = -1$. So $(0,0)$, $(1,1)$, and $(-1,1)$ are the points of intersection of given equations. Thus

$A = \int_{-1}^{1}(x^2 - x^4)dx$

$= (\frac{1}{3}x^3 - \frac{1}{5}x^5)|_{-1}^{1} = \frac{4}{15}$



31. $y = |x|$, $y = x^2 - 2$

Sol. $|x| = x^2 - 2$

if $x \geq 0$, then $x = x^2 - 2 \Rightarrow x^2 - x - 2 = 0$

$\Rightarrow (x-2)(x+1) = 0 \Rightarrow x = 2$, ($x = -1$ is not in its condition.)

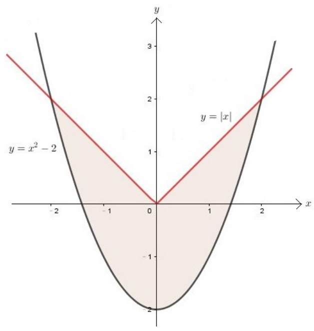
if $x < 0$, then $-x = x^2 - 2 \Rightarrow x^2 + x - 2 = 0$

$\Rightarrow (x+2)(x-1) = 0 \Rightarrow x = -2$, ($x = 1$ is not in its condition.)

So $(2,2)$, and $(-2,2)$ are the points of intersection of given equations. Thus

$A = \int_{-2}^{0}((-x) - (x^2 - 2))dx + \int_{0}^{2}(x - (x^2 - 2))dx$

$= (-\frac{1}{2}x^2 - \frac{1}{3}x^3 + 2x)|_{-2}^{0} + (\frac{1}{2}x^2 - \frac{1}{3}x^3 + 2x)|_{0}^{2} = \frac{20}{3}$



33. $y = \sin 2x$, $y = \cos x$, $x = \dfrac{\pi}{6}$, $x = \dfrac{\pi}{2}$

Sol. $A = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\sin 2x - \cos x)dx = (\frac{-1}{2}\cos 2x - \sin x)|_{\frac{\pi}{6}}^{\frac{\pi}{2}} = \frac{1}{4}$