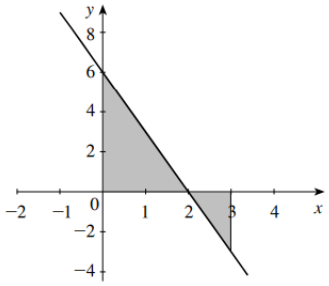
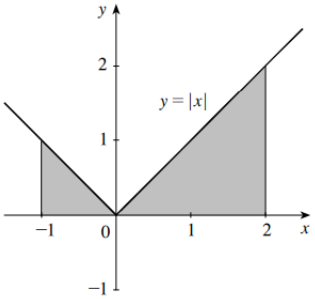
4.4 The Definite Integral

21. $\int_0^3 (-3x+6)\,dx = \frac{1}{2}(2)(6) - \frac{1}{2}(1)(3) = \frac{9}{2}$



22. $\int_{-1}^{2} |x|\,dx = \frac{1}{2}(1)(1) + \frac{1}{2}(2)(2) = \frac{5}{2}$



37. Since $\cos x \le 1$ on $\left[0, \frac{\pi}{4}\right]$, we see that $\sin^2 x \cos x\,dx \le \sin^2 x\,dx$. Therefore, by Property 5 of the definite integral, $\int_0^{\pi/4} \sin^2 x \cos x\,dx \le \int_0^{\pi/4} \sin^2 x\,dx$.

48. $-|f(x)| \le f(x) \le |f(x)|$ for all x in $[a,b]$, so by Property 5 of the definite integral,
$-\int_a^b |f(x)|\,dx = \int_a^b \left[-|f(x)|\right] dx \le \int_a^b f(x)\,dx \le \int_a^b |f(x)|\,dx \Rightarrow \left|\int_a^b f(x)\,dx\right| \le \int_a^b |f(x)|\,dx.$

4.5 The Fundamental Theorem of Calculus

8. $G'(x) = \dfrac{d}{dx}\int_0^{x^2} t \sin t\,dt = \left(x^2 \sin x^2\right)\dfrac{d}{dx}\left(x^2\right) = 2x^3 \sin x^2$

17. $\int_{-2}^{1} (3t+2)^2\,dt = \int_{-2}^{1} \left(9t^2 + 12t + 4\right) dt = 3t^3 + 6t^2 + 4t\Big|_{-2}^{1} = (3+6+4) - (-24+24-8) = 21$

34. Let $u = t+1$, so $du = dt$, $t = 0 \Rightarrow u = 1$, and $t = 2 \Rightarrow u = 3$. Then
$\int_0^2 (t+1)^{0.2}\,dt = \int_1^3 u^{0.2}\,du = \frac{1}{1.2}u^{1.2}\Big|_1^3 = \frac{1}{1.2}\left(3^{1.2} - 1\right).$

88. $\dfrac{d}{dy}\left[\int_0^x \sqrt{3+2\cos t}\,dt + \int_0^y \sin t\,dt\right] = \dfrac{d}{dy}(0) = 0 \Rightarrow \sqrt{3+2\cos x}\dfrac{dx}{dy} + \sin y = 0 \Rightarrow \dfrac{dx}{dy} = -\dfrac{\sin y}{\sqrt{3+2\cos x}}$