聯微作業解答 (2.5, 2.6)

2.5

- Find the derivative of the function.

10. $g(x) = \dfrac{\cos x}{1+x}$

Sol. $$\begin{aligned} g'(x) &= \frac{(-\sin x)(1+x) - \cos x}{(1+x)^2} \\ &= \frac{-\sin x - x\sin x - \cos x}{(1+x)^2} \end{aligned}$$

18. $y = \dfrac{\sin x \cos x}{1+\csc x}$

Sol. $$\begin{aligned} y' &= \frac{(\cos x\cos x + \sin x(-\sin x))(1+\csc x) - (\sin x\cos x)(-\csc x\cot x)}{(1+\csc x)^2} \\ &= \frac{\cos 2x(1+\csc x) + \cos x\cot x}{(1+\csc x)^2} \end{aligned}$$

20. $s = \dfrac{1-\tan t}{1+\cot t}$

Sol. $$\begin{aligned} s' &= \frac{(-\sec^2 t)(1+\cot t) - (1-\tan t)(-\csc^2 t)}{(1+\cot t)^2} \\ &= \frac{(\frac{-1}{\cos^2 t})(1+\frac{\cos t}{\sin t}) - (1-\frac{\sin t}{\cos t})(\frac{-1}{\sin^2 t})}{(1+\frac{\cos t}{\sin t})^2} \\ &= \frac{\frac{-1}{\cos^2 t} - \frac{1}{\sin t\cos t} + \frac{1}{\sin^2 t} - \frac{1}{\sin t\cos t}}{1+\frac{2\cos t}{\sin t}+\frac{\cos^2 t}{\sin^2 t}} \\ &= \frac{\frac{\cos 2t - \sin 2t}{\sin^2 t\cos^2 t}}{\frac{(\sin t+\cos t)^2}{\sin^2 t}} \\ &= \frac{\cos 2t - \sin 2t}{\cos^2 t(\sin t+\cos t)^2} \end{aligned}$$

- Find the second derivative of the function.

26. $h(t) = (t^2+1)\sin t$

Sol. $$\begin{aligned} h'(t) &= 2t\sin t + (t^2+1)\cos t \\ h''(t) &= 2\sin t + 2t\cos t + 2t\cos t + (t^2+1)(-\sin t) \\ &= 4t\cos t + (1-t^2)\sin t \\ &= 4t\cos t + \sin t - t^2\sin t \end{aligned}$$

• Find the rate of change of y with respect to x at the indicated value of x.

34. $y = \csc x - 2\cos x$; $x = \frac{\pi}{6}$

Sol. $y' = -\csc x \cot x + 2\sin x$

$y'|_{x=\frac{\pi}{6}} = (-2)\sqrt{3} + 2(\frac{1}{2}) = 1 - 2\sqrt{3}$

2.6

- Find the derivative of the function.

8. $g(x) = (3x^2 + x - 1)^{\frac{4}{3}}$

Sol. $g'(x) = \frac{4}{3}(3x^2 + x - 1)^{\frac{1}{3}}(6x + 1)$

18. $f(x) = \dfrac{x}{\sqrt{x^2 - x - 2}}$

Sol. $$\begin{aligned} f'(x) &= (x^2 - x - 2)^{-\frac{1}{2}} + x(-\tfrac{1}{2}(x^2 - x - 2)^{\frac{3}{2}}(2x - 1)) \\ &= \tfrac{1}{2}(x^2 - x - 2)^{-\frac{3}{2}}(-x(2x - 1) + 2(x^2 - x - 2)) \\ &= -\frac{x + 4}{2(x^2 - x - 2)^{\frac{3}{2}}} \end{aligned}$$

41. $f(x) = \dfrac{1 + \cos 3x}{1 - \cos 3x}$

Sol. $$\begin{aligned} f'(x) &= \frac{(-3\sin 3x)(1 - \cos 3x) - (1 + \cos 3x)(3\sin 3x)}{(1 - \cos 3x)^2} \\ &= \frac{(3\sin 3x)(-1 + \cos 3x - 1 - \cos 3x)}{(1 - \cos 3x)^2} \\ &= \frac{-6\sin 3x}{(1 - \cos 3x)^2} \end{aligned}$$

66. Suppose that $F(x) = g[f(x)]$ and $f(3) = 16$, $f'(3) = 6$, and $g'(16) = \frac{1}{8}$. Find $F'(3)$.

Sol. $F(x) = g(f(x))$

$F'(x) = g'(f(x))f'(x)$

$F'(3) = g'(f(3))f'(3) = g'(16) \cdot 6 = \frac{1}{8} \cdot 6 = \frac{3}{4}$

76. Suppose that f has second-order derivatives and $g(x) = xf(x^2+1)$. Find $g''(x)$ in terms of $f(x)$, $f'(x)$, and $f''(x)$.

Sol. $g'(x) = f(x^2+1) + xf'(x^2+1)(2x) = f(x^2+1) + 2x^2 f'(x^2+1)$

$$
\begin{aligned}
g''(x) &= f'(x^2+1)(2x) + 4xf'(x^2+1) + 2x^2 f''(x^2+1)(2x) \\
&= 6xf'(x^2+1) + 4x^3 f''(x^2+1)
\end{aligned}
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