3. $\mathbf{a}\times\mathbf{b}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\1&1&0\\0&2&3\end{vmatrix}=3\mathbf{i}-3\mathbf{j}+2\mathbf{k}$

5. $\mathbf{a}\times\mathbf{b}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\1&-2&1\\3&1&-2\end{vmatrix}=3\mathbf{i}+5\mathbf{j}+7\mathbf{k}$

15. $\mathbf{a}\times\mathbf{b}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\-3&1&-2\\1&1&1\end{vmatrix}=3\mathbf{i}+\mathbf{j}-4\mathbf{k}$, so the required unit vectors are $\pm\left(\dfrac{3\mathbf{i}+\mathbf{j}-4\mathbf{k}}{\sqrt{9+1+16}}\right)$; that is, $\pm\frac{\sqrt{26}}{26}(3\mathbf{i}+\mathbf{j}-4\mathbf{k})$.

19. For $P(1,-1,2)$, $Q(2,3,1)$, and $R(-2,3,4)$, $\overrightarrow{PQ}=\mathbf{i}+4\mathbf{j}-\mathbf{k}$ and $\overrightarrow{PR}=-3\mathbf{i}+4\mathbf{j}+2\mathbf{k}$,

so $\overrightarrow{PQ}\times\overrightarrow{PR}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\1&4&-1\\-3&4&2\end{vmatrix}=12\mathbf{i}+\mathbf{j}+16\mathbf{k}$, so the area of $\triangle PQR$ is

$\frac{1}{2}\left|\overrightarrow{PQ}\times\overrightarrow{PR}\right|=\frac{1}{2}|12\mathbf{i}+\mathbf{j}+16\mathbf{k}|=\frac{1}{2}\sqrt{144+1+256}=\frac{\sqrt{401}}{2}$.

27. $\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})=\begin{vmatrix}1&1&0\\0&1&-2\\1&2&3\end{vmatrix}=5\Rightarrow V=|\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})|=|5|=5$

29. For $P(0,0,0)$, $Q(3,-2,1)$, $R(1,2,2)$, and $S(1,1,4)$, $\overrightarrow{PQ}=\langle 3,-2,1\rangle$, $\overrightarrow{PR}=\langle 1,2,2\rangle$, and $\overrightarrow{PS}=\langle 1,1,4\rangle$, so

$\overrightarrow{PQ}\cdot\left(\overrightarrow{PR}\times\overrightarrow{PS}\right)=\begin{vmatrix}3&-2&1\\1&2&2\\1&1&4\end{vmatrix}=21$. Thus, $V=\left|\overrightarrow{PQ}\cdot\left(\overrightarrow{PR}\times\overrightarrow{PS}\right)\right|=|21|=21$.

54. Let $\mathbf{v}=a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$ be a vector in the desired direction. Since $\mathbf{v}$ lies in the plane determined by $\mathbf{a}=\mathbf{i}+3\mathbf{j}+2\mathbf{k}$ and

$\mathbf{b}=-\mathbf{i}+2\mathbf{j}+4\mathbf{k}$, $\mathbf{v}$, $\mathbf{a}$, and $\mathbf{b}$ are coplanar, so $\mathbf{v}\cdot(\mathbf{a}\times\mathbf{b})=0\Rightarrow\mathbf{v}\cdot(\mathbf{a}\times\mathbf{b})=\begin{vmatrix}a&b&c\\1&3&2\\-1&2&4\end{vmatrix}=8a-6b+5c=0$ (1). Also,

$\mathbf{v}$ is perpendicular to $\mathbf{c}=3\mathbf{i}-\mathbf{j}+2\mathbf{k}$, so $\mathbf{v}\cdot\mathbf{c}=0\Rightarrow\mathbf{v}\cdot\mathbf{c}=(a\mathbf{i}+b\mathbf{j}+c\mathbf{k})\cdot(3\mathbf{i}-\mathbf{j}+2\mathbf{k})=3a-b+2c=0$ (2). Solving

the system $\left.\begin{array}{r}8a-6b+5c=0\\3a-\ b+2c=0\end{array}\right\}$ for a and b in terms of c gives $a=-\frac{7}{10}c$ and $b=-\frac{1}{10}c$, so $\mathbf{v}=-\frac{7}{10}c\mathbf{i}-\frac{1}{10}c\mathbf{j}+c\mathbf{k}$.

We may take $c=-10$. Then the vector $\mathbf{w}=7\mathbf{i}+\mathbf{j}-10\mathbf{k}$ also satisfies the requirements, and the desired unit vector is

$\mathbf{u}=\dfrac{\mathbf{w}}{|\mathbf{w}|}=\frac{1}{5\sqrt{6}}\mathbf{w}=\frac{\sqrt{6}}{30}(7\mathbf{i}+\mathbf{j}-10\mathbf{k})=\frac{7\sqrt{6}}{30}\mathbf{i}+\frac{\sqrt{6}}{30}\mathbf{j}-\frac{\sqrt{6}}{3}\mathbf{k}$.

55. If $\mathbf{a} = \langle -1, a, 3 \rangle$ and $\mathbf{b} = \langle 2, 3, b \rangle$ are parallel, then $\mathbf{a} \times \mathbf{b} = \mathbf{0} \Leftrightarrow$

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & a & 3 \\ 2 & 3 & b \end{vmatrix} = (ab - 9)\,\mathbf{i} - (-b - 6)\,\mathbf{j} + (-3 - 2a)\,\mathbf{k} = 0\mathbf{i} + 0\mathbf{j} + 0\mathbf{k} \Leftrightarrow ab - 9 = 0,\ b + 6 = 0, \text{ and}$$

$-3 - 2a = 0$. Solving, we find $a = -\frac{3}{2}$ and $b = -6$.

11.5 Lines and Planes in Space ET 10.5

3. Parametric equations: $x = 1 + 2t$, $y = 3 + 4t$, $z = 2 + 5t$. Symmetric equations: $\dfrac{x-1}{2} = \dfrac{y-3}{4} = \dfrac{z-2}{5}$.

7. Let $A\,(2, 1, 4)$ and $B\,(1, 3, 7)$. Then $\mathbf{v} = \overrightarrow{AB} = \langle -1, 2, 3 \rangle$ is parallel to the required line. Thus, parametric equations of the line are $x = 2 - t$, $y = 1 + 2t$, $z = 4 + 3t$ and symmetric equations are $\dfrac{x-2}{-1} = \dfrac{y-1}{2} = \dfrac{z-4}{3}$.

14. Suppose L_1 and L_2 intersect at the point $P_0\,(x_0, y_0, z_0)$. Then there exist numbers t_1 and t_2 such that

$$\left.\begin{aligned} 4 + t_1 &= 6 + 2t_2 \\ 5 + t_1 &= 11 + 4t_2 \\ -1 + 2t_1 &= -3 + t_2 \end{aligned}\right\}$$

Solving the first two equations simultaneously, we find $t_1 = -2$ and $t_2 = -2$. Substituting these values into the third equation gives $-5 = -5$, so the solution to the system is $t_1 = t_2 = -2$. Therefore the point of intersection is $(2, 3, -5)$. The required line is parallel to the vector $\mathbf{v} = 4\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$ and so parametric equations of the required line are $x = 2 + 4t$, $y = 3 + 5t$, $z = -5 + 2t$.

27. A normal to the given plane is $\mathbf{n} = \langle 2, 3, -1 \rangle$. Because the required plane is parallel to the given plane, $\mathbf{n}$ is also normal to the required plane, so an equation is $2\,(x - 3) + 3\,(y - 6) - 1\,(z + 2) = 0 \Leftrightarrow 2x + 3y - z = 26$.

31. Let $A\,(1, 0, -2)$, $B\,(1, 3, 2)$, and $C\,(2, 3, 0)$. Then $\overrightarrow{AB} = \langle 0, 3, 4 \rangle$ and $\overrightarrow{AC} = \langle 1, 3, 2 \rangle$. A normal to the required plane is

$$\mathbf{n} = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 3 & 4 \\ 1 & 3 & 2 \end{vmatrix} = \langle -6, 4, -3 \rangle, \text{ so an equation of the plane is } -6\,(x - 1) + 4\,(y - 0) - 3\,(z + 2) = 0 \Leftrightarrow$$

$6x - 4y + 3z = 0$.

33. Let $P\,(1, 3, 2)$. Setting $t = 0$ gives the point $Q\,(1, -1, 3)$ on the line. Also, a vector in the same direction as the line is

$$\mathbf{v} = \langle 1, -2, 2 \rangle, \text{ so a vector normal to the required plane is } \mathbf{n} = \overrightarrow{PQ} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -4 & 1 \\ 1 & -2 & 2 \end{vmatrix} = -6\mathbf{i} + \mathbf{j} + 4\mathbf{k}, \text{ and an equation of}$$

the plane is $-6\,(x - 1) + 1\,(y - 3) + 4\,(z - 2) = 0 \Leftrightarrow 6x - y - 4z = -5$.

37. Let $P(2,1,1)$ and $Q(-1,3,2)$, so $\overrightarrow{PQ} = \langle -3,2,1\rangle$. If $\mathbf{n} = \langle 2,3,-4\rangle$ is normal to the given plane, then a normal to the required plane is $\overrightarrow{PQ} \times \mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & 2 & 1 \\ 2 & 3 & -4 \end{vmatrix} = -11\mathbf{i} - 10\mathbf{j} - 13\mathbf{k} = -(11\mathbf{i} + 10\mathbf{j} + 13\mathbf{k})$. Thus, an equation of the required plane is $11(x-2) + 10(y-1) + 13(z-1) = 0 \Leftrightarrow 11x + 10y + 13z = 45$.

43. A normal to the plane $x + y + 2z = 6$ is $\mathbf{n} = \langle 1,1,2\rangle$ and a vector parallel to the line $L: x = 1+t, y = 2+t, z = -1+t$ is $\mathbf{v} = \langle 1,1,1\rangle$, so the angle between the normal to the plane and the line is $\theta = \cos^{-1}\left(\frac{|\mathbf{n}\cdot\mathbf{v}|}{|\mathbf{n}|\,|\mathbf{v}|}\right) = \cos^{-1}\left(\frac{|\langle 1,1,2\rangle \cdot \langle 1,1,1\rangle|}{\sqrt{1+1+4}\sqrt{1+1+1}}\right) = \cos^{-1}\frac{4}{\sqrt{6}\sqrt{3}} \approx 19.5^\circ$. Therefore, the required angle is about $90^\circ - 19.5^\circ = 70.5^\circ$.

47. A normal to the plane $2x + 4y - 3z = 4$ is $\mathbf{n} = \langle 2,4,-3\rangle$. Since the line is perpendicular to the plane, its direction is given by $\mathbf{v} = \mathbf{n}$, so parametric equations of the line are $x = 2+2t, y = 3+4t, z = -1-3t$.

50. The line of intersection of the two planes is defined by the system $\left.\begin{aligned} x - y + 2z &= 1 \\ 2x + 3y - z &= 2 \end{aligned}\right\}$ To find two distinct points on the line, we take $z = 0$ and $z = 1$ successively to obtain $\left.\begin{aligned} x - y &= 1 \\ 2x + 3y &= 2 \end{aligned}\right\}$ and $\left.\begin{aligned} x - y &= -1 \\ 2x + 3y &= 3 \end{aligned}\right\}$ Solving each system, we find the points to be $P(1,0,0)$ and $Q(0,1,1)$. Letting $R(3,4,1)$ denote the given point, we see that a normal to the required plane is $\overrightarrow{RP} \times \overrightarrow{RQ} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & -4 & -1 \\ -3 & -3 & 0 \end{vmatrix} = -3\mathbf{i} + 3\mathbf{j} - 6\mathbf{k} = -3(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$. Taking $\mathbf{n} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $R(3,4,1)$, we find an equation of the required plane to be $(x-3) - (y-4) + 2(z-1) = 0 \Leftrightarrow x - y + 2z = 1$.

55. To find the distance between $P(3,1,2)$ and the plane $2x - 3y + 4z = 7 \Leftrightarrow 2x - 3y + 4z - 7 = 0$, we write $\mathbf{n} = \langle a,b,c\rangle = \langle 2,-3,4\rangle \Rightarrow D = \frac{|2(3) - 3(1) + 4(2) - 7|}{\sqrt{2^2+3^2+4^2}} = \frac{4\sqrt{29}}{29}$.