48. $f'(x) = \dfrac{d}{dx}\left(2x^4 - \frac{8}{3}x^3 - 8x^2 + 12\right) = 8x^3 - 8x^2 - 16x = 8x\left(x^2 - x - 2\right) = 8x(x-2)(x+1)$, so f has critical numbers -1, 0, and 2 on the interval $(-2, 3)$. We calculate $f(-2) = \frac{100}{3}$, $f(-1) = \frac{26}{3}$, $f(0) = 12$, $f(2) = -\frac{28}{3}$, and $f(3) = 30$, so f has an absolute maximum value of $\frac{100}{3}$ attained at $x = -2$ and an absolute minimum value of $-\frac{28}{3}$ attained at $x = 2$.

50. $g'(u) = \dfrac{d}{du}\left(\dfrac{u^{1/2}}{u^2+1}\right) = \dfrac{\left(u^2+1\right)\frac{1}{2}u^{-1/2} - u^{1/2}(2u)}{\left(u^2+1\right)^2} = \dfrac{\frac{1}{2}u^{-1/2}\left(u^2+1-4u^2\right)}{\left(u^2+1\right)^2} = \dfrac{1-3u^2}{2u^{1/2}\left(u^2+1\right)^2} = 0 \Leftrightarrow$ $u = \pm\frac{\sqrt{3}}{3}$, and we see that g has the critical number $\frac{\sqrt{3}}{3}$ on the interval $(0, 2)$. $g(0) = 0$, $g\left(\frac{\sqrt{3}}{3}\right) = \frac{\sqrt{3\sqrt{3}}}{4} \approx 0.57$, and $g(2) = \frac{\sqrt{2}}{5} \approx 0.28$, so g has an absolute minimum value of 0 attained at $x = 0$ and an absolute maximum value of $\frac{\sqrt{3\sqrt{3}}}{4}$ attained at $x = \frac{\sqrt{3}}{3}$.

58. $g'(x) = \dfrac{d}{dx}(\cos x - \sin x) = -\sin x - \cos x$ is continuous everywhere and has zeros where $-\sin x - \cos x = 0 \Leftrightarrow$ $\tan x = -1 \Leftrightarrow x = \frac{3\pi}{4}$ or $x = \frac{7\pi}{4}$ on the interval $(0, 2\pi)$. $g(0) = 1$, $g\left(\frac{3\pi}{4}\right) = -\sqrt{2}$, $g\left(\frac{7\pi}{4}\right) = \sqrt{2}$, and $g(2\pi) = 1$, so g has an absolute minimum value of $-\sqrt{2}$ attained at $x = \frac{3\pi}{4}$, and an absolute maximum value of $\sqrt{2}$ attained at $x = \frac{7\pi}{4}$.

60. $f'(x) = \dfrac{d}{dx}(x - \sin x) = 1 - \cos x$ is continuous everywhere and has zeros where $1 - \cos x = 0 \Leftrightarrow \cos x = 1 \Rightarrow$ $x = 2n\pi$, n an integer. Since none of these lies on $(0, 2\pi)$, f has no critical number in that interval. $f(0) = 0$ and $f(2\pi) = 2\pi$, so f has an absolute minimum value of 0 attained at $x = 0$ and an absolute maximum value of 2π attained at $x = 2\pi$.

22. $f(x) = 1 - x^{2/3} \Rightarrow f'(x) = -\frac{2}{3}x^{-1/3} = -\dfrac{2}{3x^{1/3}}$. Suppose there is a number c satisfying $-1 < c < 8$ such that $f'(c) = \dfrac{f(b) - f(a)}{b - a} = \dfrac{f(8) - f(-1)}{8 - (-1)}$. Then $-\dfrac{2}{3c^{1/3}} = \dfrac{\left(1 - 8^{2/3}\right) - (1 - 1)}{9} = -\dfrac{1}{3} \Leftrightarrow c^{1/3} = 2 \Leftrightarrow c = 8$. This contradiction shows that no such c exists. The result does not contradict the Mean Value Theorem because f is not differentiable on the interval $(-1, 8)$.

24. $g(x) = x^4 - 2x^2 + x$ is continuous on $[0, 1]$ and differentiable on $(0, 1)$. Furthermore, $g(0) = 0 = g(1)$. Therefore, by Rolle's Theorem, there exists at least one number c in $(0, 1)$ such that $g'(c) = 4c^3 - 4c + 1 = 0$. But $g'(x) = f(x)$, and so $f(c) = 0$, showing that f has at least one zero in $(0, 1)$.

33. Let $f(x) = \cos^2 x - \frac{1}{2}\cos 2x$. Observe that f is continuous and differentiable on $(-\infty, \infty)$. $f'(x) = 2\cos x(-\sin x) - \frac{1}{2}(-\sin 2x)(2) = -2\sin x\cos x + \sin 2x = -\sin 2x + \sin 2x = 0$. Using Theorem 3, we see that $f(x) = C$ where C is a constant; that is, $f(x) = \cos^2 x - \frac{1}{2}\cos 2x = C$. To find C, we calculate $f\left(\frac{\pi}{2}\right) = 0 - \frac{1}{2}(-1) = C \Leftrightarrow C = \frac{1}{2}$, so $\cos^2 x - \frac{1}{2}\cos 2x = \frac{1}{2} \Leftrightarrow \cos^2 x = \frac{1}{2}(1 + \cos 2x)$.

36. $f(x) = 2(x-1)(x-2)(x-3)(x-4)$ is a polynomial function, and thus continuous and differentiable on $(-\infty, \infty)$. Furthermore, $f(1) = f(2) = f(3) = f(4) = 0$. Therefore, by Rolle's Theorem, there exists at least one number c_1 in $(1, 2)$, at least one number c_2 in $(2, 3)$, and at least one number c_3 in $(3, 4)$ such that $f'(c_1) = f'(c_2) = f'(c_3) = 0$. Therefore, f' has at least three real zeros. On the other hand, f' is a polynomial of degree three and can have at most three real zeros. Therefore, f' has exactly three zeros.