4. 1

6. $\int\left(2x^{2/3}-4x^{1/3}+4\right)dx=\frac{6}{5}x^{5/3}-3x^{4/3}+4x+C$

15. $\int\frac{x^2-2x+3}{\sqrt{x}}\,dx=\int\left(x^{3/2}-2x^{1/2}+3x^{-1/2}\right)dx=\frac{2}{5}x^{5/2}-\frac{4}{3}x^{3/2}+6x^{1/2}+C$

25. $\int\frac{1-2\cot^2 x}{\cos^2 x}\,dx=\int\left(\sec^2 x-2\csc^2 x\right)dx=\tan x+2\cot x+C$

37. $f(x)=\int f'(x)\,dx=\int\frac{1}{\sqrt{x}}\,dx=\int x^{-1/2}\,dx=2x^{1/2}+C$. $f(4)=2\Rightarrow 4+C=2\Rightarrow C=-2$. Thus, $f(x)=2\sqrt{x}-2$.

42. $f'(x)=\int f''(x)\,dx=\int(2x+1)\,dx=x^2+x+C_1$. $f'(0)=1\Rightarrow 0+C_1=1$, so $f'(x)=x^2+x+1$. Then $f(x)=\int f'(x)\,dx=\int\left(x^2+x+1\right)dx=\frac{1}{3}x^3+\frac{1}{2}x^2+x+C_2$. $f(0)=5\Rightarrow 0+C_2=5$ or $C_2=5$. Thus, $f(x)=\frac{1}{3}x^3+\frac{1}{2}x^2+x+5$.

4.2

15. For $I = \int \sqrt{1-2x}\,dx$, let $u = 1-2x \Rightarrow du = -2\,dx \Rightarrow dx = -\frac{1}{2}\,du$. Then

$\int \sqrt{1-2x}\,dx = -\frac{1}{2}\int \sqrt{u}\,du = -\frac{1}{2}\left(\frac{2}{3}u^{3/2}\right) + C = -\frac{1}{3}\sqrt{(1-2x)^3} + C.$

41. For $I = \displaystyle\int \frac{\sec x \tan x}{(\sec x - 1)^2}\,dx$, let $u = \sec x - 1 \Rightarrow du = \sec x \tan x\,dx$. Then

$I = \displaystyle\int \frac{du}{u^2} = -\frac{1}{u} + C = -\frac{1}{\sec x - 1} + C = \frac{1}{1 - \sec x} + C.$

44. For $I = \displaystyle\int \frac{1+\sin x}{\cos^2 x}\,dx = \int \left(\frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x}\right)dx = \int \sec^2 x\,dx + \int \frac{\sin x}{\cos^2 x}\,dx$, let $u = \cos x$ in the second integral.

Then $du = -\sin x\,dx$, so $I = \displaystyle\int \sec^2 x\,dx - \int \frac{du}{u^2} = \tan x + \frac{1}{u} + C = \tan x + \frac{1}{\cos x} + C = \tan x + \sec x + C.$

45. For $I = \int x\sqrt{x-4}\,dx$, let $u = x - 4 \Rightarrow du = dx$ and $x = u + 4$. Then

$I = \int (u+4)\sqrt{u}\,du = \int \left(u^{3/2} + 4u^{1/2}\right)dx = \frac{2}{5}u^{5/2} + \frac{8}{3}u^{3/2} + C$

$= \frac{2}{15}u^{3/2}(3u+20) + C = \frac{2}{15}(3x+8)\sqrt{(x-4)^3} + C.$

49. For $I = \displaystyle\int \frac{dx}{\sqrt{x} + \sqrt{x+1}} = \int \frac{1}{\sqrt{x}+\sqrt{x+1}} \cdot \frac{\sqrt{x}-\sqrt{x+1}}{\sqrt{x}-\sqrt{x+1}}\,dx = \int \frac{\sqrt{x}-\sqrt{x+1}}{-1}\,dx = \int \sqrt{x+1}\,dx - \int \sqrt{x}\,dx$, let

$u = x + 1 \Rightarrow du = dx$ in the first integral. Then

$I = \int u^{1/2}\,du - \int x^{1/2}\,dx = \frac{2}{3}u^{3/2} - \frac{2}{3}x^{3/2} + C = \frac{2}{3}(x+1)^{3/2} - \frac{2}{3}x^{3/2} + C.$