Calculus I

2016/10/18

考試注意事項：

1. 答案紙直行對折，兩直欄書寫作答。
2. 無清楚計算過程，不予計分。

試題：

1. (15%) Find $\dfrac{dy}{dx}$ if $y = \tan^3(3x^2 + 1)$.
2. (15%) Find $\dfrac{dy}{dx}$ at the point $(\frac{\pi}{2}, \pi)$ if $x \sin y - y \cos 2x = 2x$.
3. (15%) Find the extreme value of the function $f(x) = 3x^4 - 4x^3 - 8$ on $[-1, 2]$.
4. (15%) Find an equation of the tangent line to the graph of
$$f(x) = \frac{(2x^2 + 1)(x^3 - 1)}{x^2 + 4}.$$
5. (10%) Find $\lim\limits_{x \to 0} \dfrac{\tan x}{x}$.
6. (10%) Let
$$f(x) = \begin{cases} x + 2 & \text{if } x \leq 1, \\ kx^2 & \text{if } x > 1. \end{cases}$$
Find the value of k that will make f continuous on $(-\infty, \infty)$.
7. (10%) Find the points on the graph of $f(x) = x^4 - 2x^2 + 2$ where the tangent line is horizontal.
8. (10%) Prove that $f(x) = 4x^3 - 4x + 1$ has at least one zero in the interval $(0, 1)$.

   Hint: Apply Roll's theorem to the function $g(x) = x^4 - 2x^2 + x$ on $[0, 1]$.