12.1

3. $f(t) = \sqrt{t}$ is defined for $t \geq 0$, $g(t) = \dfrac{1}{t-1}$ is not defined at 1, and $h(t) = \ln t$ is defined for $t > 0$, so the domain of $\mathbf{r}$ is $(0, 1) \cup (1, \infty)$.

6. $f(t) = \sqrt[3]{t}$ is defined on $(-\infty, \infty)$, $g(t) = e^{1/t}$ is defined on $(-\infty, 0) \cup (0, \infty)$, and $h(t) = \dfrac{1}{t+2}$ is defined on $(-\infty, -2) \cup (-2, \infty)$, so the domain of $\mathbf{r}$ is $(-\infty, -2) \cup (-2, 0) \cup (0, \infty)$.

33. $x + y + 2z = 1 \Rightarrow z = \frac{1}{2}(1 - x - y)$, but $x^2 + y^2 = 1$ can be parametrized by $x = \cos t$, $y = \sin t$, $0 \leq t \leq 2\pi$, so the curve is described by $\mathbf{r}(t) = \left\langle \cos t, \sin t, \dfrac{1 - \cos t - \sin t}{2} \right\rangle$, $0 \leq t \leq 2\pi$.

39. $\displaystyle \lim_{t\to 2}\left[\sqrt{t}\,\mathbf{i} + \frac{t^2-4}{t-2}\mathbf{j} + \frac{t}{t^2+1}\mathbf{k}\right] = \lim_{t\to 2}\sqrt{t}\,\mathbf{i} + \lim_{t\to 2}(t+2)\,\mathbf{j} + \lim_{t\to 2}\frac{t}{t^2+1}\mathbf{k} = \sqrt{2}\,\mathbf{i} + 4\mathbf{j} + \tfrac{2}{5}\mathbf{k}$

41. $\displaystyle \lim_{t\to\infty}\left\langle e^{-t}, \frac{1}{t}, \frac{2t^2}{t^2+1}\right\rangle = \left\langle \lim_{t\to\infty} e^{-t}, \lim_{t\to\infty}\frac{1}{t}, \lim_{t\to\infty}\frac{2}{1+(1/t^2)}\right\rangle = \langle 0, 0, 2\rangle$

42. $\displaystyle \lim_{t\to -\infty}\left[\frac{t-1}{2t+1}\mathbf{i} + e^{2t}\mathbf{j} + \tan^{-1} t\,\mathbf{k}\right] = \lim_{t\to-\infty}\frac{1-\frac{1}{t}}{2+\frac{1}{t}}\mathbf{i} + \lim_{t\to-\infty} e^{2t}\mathbf{j} + \lim_{t\to-\infty}\tan^{-1} t\,\mathbf{k} = \tfrac{1}{2}\mathbf{i} - \tfrac{\pi}{2}\mathbf{k}$

45. Since $f(t) = \dfrac{\cos t - 1}{t}$ has domain $(-\infty, 0) \cup (0, \infty)$, $g(t) = \dfrac{\sqrt{t}}{1+2t}$ is continuous on $[0, \infty)$, and $h(t) = te^{-1/t}$ is continuous on $(-\infty, 0)$ and $(0, \infty)$, we see that $\mathbf{r}$ is continuous on $(0, \infty)$.

47. Since $f(t) = e^{-t}$ is continuous on $(-\infty, \infty)$, $g(t) = \cos\sqrt{4-t}$ is continuous on $(-\infty, 4]$, and $h(t) = 1/\left(t^2 - 1\right)$ is continuous on $(-\infty, -1)$, $(-1, 1)$, and $(1, \infty)$, we see that $\mathbf{r}$ is continuous on $(-\infty, -1)$, $(-1, 1)$, and $(1, 4]$.

12. 2

4. $\mathbf{r}(t) = \langle t\cos t, t\sin t, \tan t\rangle \Rightarrow \mathbf{r}'(t) = \left\langle \cos t - t\sin t, \sin t + t\cos t, \sec^2 t\right\rangle \Rightarrow$

$\mathbf{r}''(t) = \left\langle -t\cos t - 2\sin t, 2\cos t - t\sin t, 2\sec^2 t\tan t\right\rangle$

6. $\mathbf{r}(t) = e^{-t}\mathbf{i} + te^{t}\mathbf{j} + e^{-2t}\mathbf{k} \Rightarrow \mathbf{r}'(t) = -e^{-t}\mathbf{i} + (t+1)e^{t}\mathbf{j} - 2e^{-2t}\mathbf{k} \Rightarrow \mathbf{r}''(t) = e^{-t}\mathbf{i} + (t+2)e^{t}\mathbf{j} + 4e^{-}$

7. $\mathbf{r}(t) = e^{-t}\sin t\,\mathbf{i} + e^{-t}\cos t\,\mathbf{j} + \tan^{-1} t\,\mathbf{k} \Rightarrow \mathbf{r}'(t) = (\cos t - \sin t)e^{-t}\mathbf{i} - (\cos t + \sin t)e^{-t}\mathbf{j} + \dfrac{1}{t^2+1}\mathbf{k} \Rightarrow$

$\mathbf{r}''(t) = -2e^{-t}\cos t\,\mathbf{i} + 2e^{-t}\sin t\,\mathbf{j} - \dfrac{2t}{(t^2+1)^2}\mathbf{k}$

8. $\mathbf{r}(t) = \left\langle \sin^{-1} t, \sec t, \ln|t|\right\rangle \Rightarrow \mathbf{r}'(t) = \left\langle \dfrac{1}{\sqrt{1-t^2}}, \sec t\tan t, \dfrac{1}{t}\right\rangle \Rightarrow \mathbf{r}''(t) = \left\langle \dfrac{t}{(1-t^2)^{3/2}}, \sec t\left(\sec^2 t + \tan^2 t\right), -\dfrac{1}{t^2}\right\rangle$

17. $\mathbf{r}(t) = t\mathbf{i} + 2t\mathbf{j} + 3t\mathbf{k} \Rightarrow \mathbf{r}'(t) = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \Rightarrow \mathbf{r}'(1) = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$, so $|\mathbf{r}'(1)| = \sqrt{1^2+2^2+3^2} = \sqrt{14}$, and therefore

$\mathbf{T}(1) = \dfrac{\mathbf{r}'(1)}{|\mathbf{r}'(1)|} = \dfrac{\mathbf{i}+2\mathbf{j}+3\mathbf{k}}{\sqrt{14}} = \frac{\sqrt{14}}{14}\mathbf{i} + \frac{\sqrt{14}}{7}\mathbf{j} + \frac{3\sqrt{14}}{14}\mathbf{k}.$

19. $\mathbf{r}(t) = 2\sin 2t\,\mathbf{i} + 3\cos 2t\,\mathbf{j} + 3\mathbf{k} \Rightarrow \mathbf{r}'(t) = 4\cos 2t\,\mathbf{i} - 6\sin 2t\,\mathbf{j} \Rightarrow \mathbf{r}'\left(\frac{\pi}{6}\right) = 2\mathbf{i} - 3\sqrt{3}\mathbf{j}$, so

$\left|\mathbf{r}'\left(\frac{\pi}{6}\right)\right| = \sqrt{2^2 + \left(-3\sqrt{3}\right)^2} = \sqrt{31}$, and therefore $\mathbf{T}\left(\frac{\pi}{6}\right) = \dfrac{\mathbf{r}'\left(\frac{\pi}{6}\right)}{\left|\mathbf{r}'\left(\frac{\pi}{6}\right)\right|} = \dfrac{1}{\sqrt{31}}\left(2\mathbf{i} - 3\sqrt{3}\mathbf{j}\right) = \frac{2\sqrt{31}}{31}\mathbf{i} - \frac{3\sqrt{93}}{31}\mathbf{j}.$

26. $\mathbf{r}(t) = e^{-t}\cos t\,\mathbf{i} + e^{-t}\sin t\,\mathbf{j} + \sin^{-1} t\,\mathbf{k} \Rightarrow \mathbf{r}'(t) = -e^{-t}(\sin t + \cos t)\mathbf{i} - e^{-t}(\sin t - \cos t)\mathbf{j} + \dfrac{1}{\sqrt{1-t^2}}\mathbf{k}$, so a vector equation of the tangent line when $t = 0$ is $\mathbf{r}'(t) = \mathbf{r}(0) + t\mathbf{r}'(0) = \mathbf{i} + t(-\mathbf{i}+\mathbf{j}+\mathbf{k}) = (1-t)\mathbf{i} + t\mathbf{j} + t\mathbf{k}$ and parametric equations are $x = 1-t, y = t, z = t$.

30. $\int_1^2 \left[\sqrt{t-1}\mathbf{i} + \frac{1}{\sqrt{t}}\mathbf{j} + (2t-1)^5\mathbf{k}\right]dt = \left[\frac{2}{3}(t-1)^{3/2}\mathbf{i} + 2t^{1/2}\mathbf{j} + \frac{1}{12}(2t-1)^6\mathbf{k}\right]_1^2 = \left(\frac{2}{3}\mathbf{i} + 2\sqrt{2}\mathbf{j} + \frac{243}{4}\mathbf{k}\right) -$
$\left(2\mathbf{j} + \frac{1}{12}\mathbf{k}\right) = \frac{2}{3}\mathbf{i} + 2\left(\sqrt{2}-1\right)\mathbf{j} + \frac{182}{3}\mathbf{k}$

31. $\int(\sin 2t\,\mathbf{i} + \cos 2t\,\mathbf{j} + e^{-t}\mathbf{k})\,dt = -\frac{1}{2}\cos 2t\,\mathbf{i} + \frac{1}{2}\sin 2t\,\mathbf{j} - e^{-t}\mathbf{k} + \mathbf{C}$

32. $\int\left(te^{t}\mathbf{i} + 2\mathbf{j} - \sec^2 t\,\mathbf{k}\right)dt = (t-1)e^{t}\mathbf{i} + 2t\mathbf{j} - \tan t\,\mathbf{k} + \mathbf{C}$

33. $\int\left(t\cos t\,\mathbf{i} + t\sin t^2\,\mathbf{j} - te^{t^2}\mathbf{k}\right)dt = (\cos t + t\sin t)\mathbf{i} - \frac{1}{2}\cos t^2\,\mathbf{j} - \frac{1}{2}e^{t^2} + \mathbf{C}$. (Note that we have integrated $\int t\cos t\,dt$ by parts.)

34. $\int\left(\dfrac{1}{1+t^2}\mathbf{i} + \dfrac{t}{1+2t^2}\mathbf{j} - \dfrac{1}{\sqrt{1-t^2}}\mathbf{k}\right)dt = \tan^{-1} t\,\mathbf{i} + \frac{1}{4}\ln\left(2t^2+1\right)\mathbf{j} - \sin^{-1} t\,\mathbf{k} + \mathbf{C}$

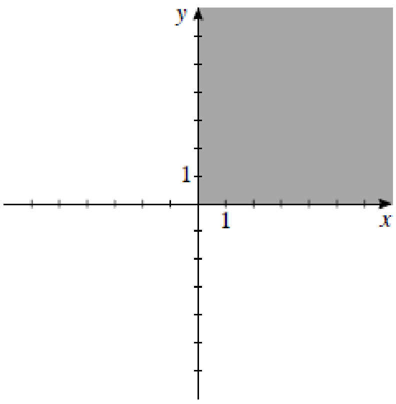
13. 1

5. Since $f(x, y) = x + 3y - 1$ is defined for all pairs (x, y) of real numbers, the domain of f is $\{(x, y) \mid -\infty < x < \infty, -\infty < y < \infty\}$. The range of f is $\{z \mid -\infty < z < \infty\}$.

7. Since $u \neq v$, the domain of f is $\{(u, v) \mid u \neq v\}$. The range of f is $\{z \mid -\infty < z < \infty\}$.

9. We require that the radicand $4 - x^2 - y^2 \geq 0 \Leftrightarrow x^2 + y^2 \leq 4$, so the domain of g is $\left\{(x, y) \mid x^2 + y^2 \leq 4\right\}$. The range of g is $\{z \mid 0 \leq z \leq 2\}$.

15. The domain of f is $\{(x, y) \mid x \geq 0 \text{ and } y \geq 0\}$



17. The domain of f is $\{(u, v) \mid u \neq v, u \neq -v\}$