聯合微積分 作業解答 5.2 5.3

5.2 Volumes:Disks, Washers, and Cross Sections

Find the volume of the solid generated by revolving the region bounded by the graphs of the equations and /or inequalities about the indicated axis.

15. $y = -x^2 + 2x, y = 0$; the x-axis

$$-x^2 + 2x = 0 \Rightarrow x = 0 \ or \ 2$$

$$\begin{aligned}
\int_0^2 \pi(-x^2+2x)^2 dx &= \pi \int_0^2 x^4 - 4x^3 + 4x^2 dx \\
&= \pi(\frac{1}{5}x^5 - x^4 + \frac{4}{3}x^3)|_0^2 \\
&= \pi(\frac{32}{5} - 16 + \frac{32}{3}) \\
&= \frac{16}{15}\pi
\end{aligned}$$

16. $y = \sqrt{x-1}, y = 0, x = 2, x = 5$; the x-axis

$$\begin{aligned}
\int_2^5 \pi(\sqrt{x-1})^2 dx &= \pi \int_2^5 x - 1 dx \\
&= \pi(\frac{x^2}{2} - x)|_2^5 \\
&= \pi[(\frac{25}{2} - 5) - (\frac{4}{2} - 2)] \\
&= \frac{15}{2}\pi
\end{aligned}$$

Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the indicated line.

35. $y = 4 - x^2$, $y = 0$; the line $y = 5$

$$4 - x^2 = 0 \Rightarrow x = 2 \ or \ -2$$

$$\begin{aligned}\int_{-2}^{2} \pi(0-5)^2 - \pi[(4-x^2)-5]^2 dx &= \pi \int_{-2}^{2} (4-x^2)(6+x^2) dx \\ &= \pi \int_{-2}^{2} 24 - 2x^2 - x^4 dx \\ &= \pi(24x - \frac{2}{3}x^3 - \frac{1}{5}x^5)|_{-2}^{2} \\ &= \pi(96 - \frac{32}{3} - \frac{64}{5}) \\ &= \frac{1088}{15}\pi\end{aligned}$$

37. $x = y^2 - 4y + 5$, $x = 2$; the line $x = -1$

$$y^2 - 4y + 5 = 2 \Rightarrow (y-3)(y-1) = 0$$

$$\begin{aligned}\int_{1}^{3} \pi[2-(-1)]^2 - \pi[(y^2-4y+5)-(-1)]^2 dy &= \pi \int_{1}^{3} 9 - (y^2 - 4y + 6)^2 dy \\ &= \pi \int_{1}^{3} (-y^4 + 8y^3 - 28y^2 + 48y - 27) dy \\ &= \pi(\frac{-1}{5}y^5 + 2y^4 - \frac{28}{3}y^3 + 24y^2 - 27y)|_{1}^{3} \\ &= \frac{104}{15}\pi\end{aligned}$$

Find the volume of the solid with the given base R and the indicated shape of every cross section taken perpendicular to the x-axis .

53. Cross section : a square

$$A(x) = \pi x$$

$$\begin{aligned}\int_{0}^{4} A(x) dx &= \int_{0}^{4} \pi x dx \\ &= 8\pi\end{aligned}$$

5.3 Volumes Using Cylindrical Shells

5. Use the method of cylindrical shells to find the volume of the solid generated by revolving the region about the indicated axis or line.

$$
\begin{aligned}
y^2 = 8x, y = x^2 &\Rightarrow (y^2)^2 = 64y \\
&\Rightarrow y(y-4)(y^2+4y+16) = 0
\end{aligned}
$$

$$
\begin{aligned}
\int_0^4 2\pi y(\sqrt{y} - \frac{1}{8}y^2)dy &= 2\pi \int_0^4 y^{\frac{3}{2}} - \frac{1}{8}y^3 dy \\
&= 2\pi(\frac{2}{5}y^{\frac{5}{2}} - \frac{1}{32}y^4)|_0^4 \\
&= \frac{48}{5}\pi
\end{aligned}
$$

Use the method of cylindrical shells to find the volume of the solid generated by revolving the region bounded by the graphs of the equations and /or inequalities about the indicated axis. Sketch the region and a representative rectangle.

17. $y = \sqrt{1-x^2}$, $y = -x + 1$; the y-axis

$$
\begin{aligned}
\int_0^1 2\pi x[\sqrt{1-x^2} - (-x+1)]dx &= 2\pi \int_0^1 x\sqrt{1-x^2} + x^2 - x dx \\
&= 2\pi[\frac{-1}{3}(1-x^2)^{\frac{3}{2}} + \frac{1}{3}x^3 - \frac{1}{2}x^2]|_0^1 \\
&= 2\pi(\frac{1}{3} - \frac{1}{2} - \frac{-1}{3}) \\
&= \frac{1}{3}\pi
\end{aligned}
$$

Find the volume of the solid generated by the revolving the region bounded by the graphs of the equations about the indicated line. Sketch the region and a representative rectangle.

30. $y = x^2 + 1,\ y = 0$, $x = 0,\ x = 2$; the line $x = 3$

$$
\begin{aligned}
\int_0^2 2\pi(3-x)(x^2+1)dx &= 2\pi \int_0^2 -x^3 + 3x^2 - x + 3dx \\
&= 2\pi(\frac{-1}{4}x^4 + x^3 - \frac{1}{2}x^2 + 3x)|_0^2 \\
&= 16\pi
\end{aligned}
$$

32. $y = \sqrt{x}, y = 0, x = 4$; the line $y = 2$

$$
\begin{aligned}
\int_0^2 2\pi(2-x)(4-\sqrt{x})dx &= 2\pi(\frac{2}{5}x^{\frac{5}{2}} - 2x^2 - \frac{4}{3}x^{\frac{3}{2}} + 8x)|_0^2 \\
&= 16\pi
\end{aligned}
$$

34. $y = x,\ y = x^2$; the line $y = 2$

$$\begin{aligned}\int_0^1 2\pi(2-y)(\sqrt{y}-y)dy &= 2\pi\int_0^1 y^2 - y^{\frac{3}{2}} - 2y + 2y^{\frac{1}{2}}\,dy\\ &= 2\pi(\frac{1}{3}y^3 - \frac{2}{5}y^{\frac{5}{2}} - y^2 + \frac{4}{3}y^{\frac{3}{2}})|_0^1\\ &= \frac{8}{15}\pi\end{aligned}$$