

7.1 Integration by Parts

7. $I = \int x^2 e^{-x} dx$. Let $u = x^2$ and $dv = e^{-x} dx$. Then $du = 2x dx$ and $v = \int e^{-x} dx = -e^{-x}$, so $I = uv - \int v du = -x^2 e^{-x} + \int 2x e^{-x} dx$. To evaluate $J = \int x e^{-x} dx$, let $s = x$ and $dt = e^{-x} dx$. Then $ds = dx$ and $t = \int e^{-x} dx = -e^{-x}$, so $J = st - \int t ds = -x e^{-x} + \int e^{-x} dx = -x e^{-x} - e^{-x}$, and therefore $I = \int x^2 e^{-x} dx = -x^2 e^{-x} + 2J = -x^2 e^{-x} + 2(-x e^{-x} - e^{-x}) + C = -(x^2 + 2x + 2)e^{-x} + C$.

11. $\int \tan^{-1} x dx$. Let $u = \tan^{-1} x$ and $dv = dx$. Then $du = \frac{dx}{1+x^2}$ and $v = x$, so $\int \tan^{-1} x dx = uv - \int v du = x \tan^{-1} x - \int \frac{x dx}{1+x^2} = x \tan^{-1} x - \frac{1}{2} \ln(1+x^2) + C$.

34. $\int_0^2 \ln(x+1) dx$. Let $u = x+1$, so $du = dx$, $x=0 \Rightarrow u=1$, and $x=2 \Rightarrow u=3$. Then $\int_0^2 \ln(x+1) dx = \int_1^3 \ln u du = (u \ln u - u)|_1^3 = 3 \ln 3 - 2$. (See Example 6.)

35. $\int_0^{1/2} \cos^{-1} x dx$. Let $u = \cos^{-1} x$ and $dv = dx$, so $du = -dx/\sqrt{1-x^2}$ and $v = x$. Then $\int_0^{1/2} \cos^{-1} x dx = x \cos^{-1} x|_0^{1/2} + \int x (1-x^2)^{-1/2} dx = \frac{1}{2} \cdot \frac{\pi}{3} + \left[-\frac{1}{2} \cdot 2(1-x^2)^{1/2} \right]_0^{1/2} = \frac{\pi}{6} - \frac{\sqrt{3}}{2} + 1 = \frac{1}{6}(\pi + 6 - 3\sqrt{3})$

7.2 Trigonometric Integrals

$$7. \int_0^{\pi} \cos^2 \frac{x}{2} dx = \int_0^{\pi} \frac{1 + \cos x}{2} dx = \frac{x + \sin x}{2} \Big|_0^{\pi} = \frac{\pi}{2}$$

$$19. \int_0^{\pi/4} \tan^2 x dx = \int_0^{\pi/4} (\sec^2 x - 1) dx = (\tan x - x) \Big|_0^{\pi/4} = 1 - \frac{\pi}{4} = \frac{4-\pi}{4}$$

$$39. \int_0^{\pi/2} \sin x \cos 2x dx = \frac{1}{2} \int_0^{\pi/2} [\sin(1-2)x + \sin(1+2)x] dx = \frac{1}{2} \int_0^{\pi/2} (-\sin x + \sin 3x) dx = \frac{1}{2} \left(\cos x - \frac{1}{3} \cos 3x \right) \Big|_0^{\pi/2} = \frac{1}{2} \left(0 - \frac{2}{3} \right) = -\frac{1}{3}$$

$$47. \int \frac{1 - \tan^2 x}{\sec^2 x} dx = \int \frac{1 - (\sec^2 x - 1)}{\sec^2 x} dx = \int \frac{2 - \sec^2 x}{\sec^2 x} dx = \int (2 \cos^2 x - 1) dx = \int \cos 2x dx = \frac{1}{2} \sin 2x + C$$