

3. Along $y = 0$, $\lim_{(x,y) \rightarrow (0,0)} \frac{3xy}{3x^2 + y^2} = \lim_{x \rightarrow 0} \frac{0}{3x^2} = \lim_{x \rightarrow 0} 0 = 0$. Along $y = x$,
 $\lim_{(x,y) \rightarrow (0,0)} \frac{3xy}{3x^2 + y^2} = \lim_{x \rightarrow 0} \frac{3x^2}{3x^2 + x^2} = \lim_{x \rightarrow 0} \frac{3}{4} = \frac{3}{4}$. Because these two limits are not equal, the given limit does not exist.

4. Along $y = 0$, $\lim_{(x,y) \rightarrow (0,0)} \frac{xy^2}{x^2 + y^4} = \lim_{x \rightarrow 0} \frac{0}{x^2} = \lim_{x \rightarrow 0} 0 = 0$. Along $x = y^2$,
 $\lim_{(x,y) \rightarrow (0,0)} \frac{xy^2}{x^2 + y^4} = \lim_{y \rightarrow 0} \frac{y^4}{2y^4} = \lim_{y \rightarrow 0} \frac{1}{2} = \frac{1}{2}$. Because these two limits are not equal, the given limit does not exist.

$$15. \lim_{(x,y) \rightarrow (1,2)} \frac{2x^2 - 3y^3 + 4}{3 - xy} = \frac{2(1)^2 - 3(2)^3 + 4}{3 - (1)(2)} = -18$$

$$19. \lim_{(x,y) \rightarrow (0^+, 0^+)} \frac{e^{\sqrt{x+y}}}{x+y-1} = \frac{e^0}{-1} = -1$$

$$23. \lim_{(x,y) \rightarrow (2,1)} \ln(x^2 - 3y) = \ln(4 - 3) = \ln 1 = 0$$

31. The function f is a rational function and is continuous at all points where its denominator is nonzero. Thus, f is continuous on $\{(x, y) \mid 2x + 3y \neq 1\}$.

33. The function g is continuous for all (x, y) such that $x + y \geq 0$ and $x - y \geq 0$; that is, on $\{(x, y) \mid x \geq 0, |y| \leq x\}$.

35. We require that $x \geq 0$ and $y \neq 0$. Thus, F is continuous on $\{(x, y) \mid x \geq 0, y \neq 0\}$.

13.3 Partial Derivatives

$$10. f_x(x, y) = \frac{\partial}{\partial x} (2x^2 - y^3)^4 = 4(2x^2 - y^3)^3 \frac{\partial}{\partial x} (2x^2 - y^3) = 4(2x^2 - y^3)^3 (4x) = 16x(2x^2 - y^3)^3 \text{ and}$$

$$f_y(x, y) = \frac{\partial}{\partial y} (2x^2 - y^3)^4 = 4(2x^2 - y^3)^3 \frac{\partial}{\partial y} (2x^2 - y^3) = 4(2x^2 - y^3)^3 (-3y^2) = -12y^2(2x^2 - y^3)^3.$$

$$12. h_u(u, v) = \frac{\partial}{\partial u} \ln(u^2 + v^2) = \frac{\frac{\partial}{\partial u} (u^2 + v^2)}{u^2 + v^2} = \frac{2u}{u^2 + v^2} \text{ and by symmetry, } h_v(u, v) = \frac{2v}{u^2 + v^2}.$$

$$14. f_x(x, y) = \frac{\partial}{\partial x} (e^x \cos y + e^y \sin x) = e^x \cos y + e^y \cos x \text{ and } f_y(x, y) = \frac{\partial}{\partial y} (e^x \cos y + e^y \sin x) = -e^x \sin y + e^y \sin x.$$

$$20. \frac{\partial z}{\partial x} = \frac{\partial}{\partial x} [\ln(e^x + y^2)] = \frac{\frac{\partial}{\partial x} (e^x + y^2)}{e^x + y^2} = \frac{e^x}{e^x + y^2} \text{ and } \frac{\partial z}{\partial y} = \frac{\partial}{\partial y} [\ln(e^x + y^2)] = \frac{\frac{\partial}{\partial y} (e^x + y^2)}{e^x + y^2} = \frac{2y}{e^x + y^2}.$$

21. $f_x(x, y) = \frac{\partial}{\partial x} y^x = y^x \ln y$ and $f_y(x, y) = \frac{\partial}{\partial y} y^x = xy^{x-1}$.

22. $f_x(x, y) = \frac{\partial}{\partial x} \int_x^y \cos t \, dt = -\frac{\partial}{\partial x} \int_y^x \cos t \, dt = -\cos x$ and $f_y(x, y) = \frac{\partial}{\partial y} \int_x^y \cos t \, dt = \cos y$.

23. $f_x(x, y) = \frac{\partial}{\partial x} \int_x^y te^{-t} \, dt = -\frac{\partial}{\partial x} \int_y^x te^{-t} \, dt = -xe^{-x}$ and $f_y(x, y) = \frac{\partial}{\partial y} \int_x^y te^{-t} \, dt = ye^{-y}$.

31. $\frac{\partial}{\partial x} (xe^y + ye^{-x} + e^z) = \frac{\partial}{\partial x} (10) \Rightarrow e^y - ye^{-x} + e^z \frac{\partial z}{\partial x} = 0 \Rightarrow \frac{\partial z}{\partial x} = \frac{ye^{-x} - e^y}{e^z}$ and $\frac{\partial}{\partial y} (xe^y + ye^{-x} + e^z) = \frac{\partial}{\partial y} (10) \Rightarrow xe^y + e^{-x} + e^z \frac{\partial z}{\partial y} = 0 \Rightarrow \frac{\partial z}{\partial y} = -\frac{xe^y + e^{-x}}{e^z}$.

33. $\frac{\partial}{\partial x} [\ln(x^2 + z^2) + yz^3 + 2x^2] = \frac{\partial}{\partial x} (10) \Rightarrow \frac{2x + 2z \frac{\partial z}{\partial x}}{x^2 + z^2} + 3yz^2 \frac{\partial z}{\partial x} + 4x = 0 \Rightarrow \frac{\partial z}{\partial x} = -\frac{2x(2x^2 + 2z^2 + 1)}{z(3yz^3 + 3x^2yz + 2)}$ and
 $\frac{\partial}{\partial y} [\ln(x^2 + z^2) + yz^3 + 2x^2] = \frac{\partial}{\partial y} (10) \Rightarrow \frac{2z \frac{\partial z}{\partial y}}{x^2 + z^2} + z^3 + 3yz^2 \frac{\partial z}{\partial y} = 0 \Rightarrow \frac{\partial z}{\partial y} = -\frac{z^2(x^2 + z^2)}{3yz^3 + 3x^2yz + 2}$.

35. $g_x(x, y) = \frac{\partial}{\partial x} (x^3y^2 + xy^3 - 2x + 3y + 1) = 3x^2y^2 + y^3 - 2$,

$g_y(x, y) = \frac{\partial}{\partial y} (x^3y^2 + xy^3 - 2x + 3y + 1) = 2x^3y + 3xy^2 + 3$, $g_{xx}(x, y) = \frac{\partial}{\partial x} (3x^2y^2 + y^3 - 2) = 6xy^2$,

$g_{yy} = \frac{\partial}{\partial y} (2x^3y + 3xy^2 + 3) = 2x^3 + 6xy$, $g_{xy} = \frac{\partial}{\partial y} (3x^2y^2 + y^3 - 2) = 6x^2y + 3y^2$, and

$g_{yx} = \frac{\partial}{\partial x} (2x^3y + 3xy^2 + 3) = 6x^2y + 3y^2$.

37. $\frac{\partial w}{\partial u} = \frac{\partial}{\partial u} [\cos(2u - v) + \sin(2u + v)] = -2 \sin(2u - v) + 2 \cos(2u + v)$,

$\frac{\partial w}{\partial v} = \frac{\partial}{\partial v} [\cos(2u - v) + \sin(2u + v)] = \sin(2u - v) + \cos(2u + v)$, $\frac{\partial^2 w}{\partial u^2} = -4 \cos(2u - v) - 4 \sin(2u + v)$,

$\frac{\partial^2 w}{\partial v^2} = -\cos(2u - v) - \sin(2u + v)$, and $\frac{\partial^2 w}{\partial u \partial v} = \frac{\partial^2 w}{\partial v \partial u} = 2 \cos(2u - v) - 2 \sin(2u + v)$.

47. $f_x(x, y) = \frac{\partial}{\partial x} (x \sin^2 y + y^2 \cos x) = \sin^2 y - y^2 \sin x \Rightarrow f_{xy}(x, y) = \frac{\partial}{\partial y} (\sin^2 y - y^2 \sin x) = 2 \sin y \cos y - 2y \sin x$

and $f_y(x, y) = \frac{\partial}{\partial y} (x \sin^2 y + y^2 \cos x) = 2x \sin y \cos y + 2y \cos x \Rightarrow$

$f_{yx}(x, y) = \frac{\partial}{\partial x} (2x \sin y \cos y + 2y \cos x) = 2 \sin y \cos y - 2y \sin x$, so $f_{xy} = f_{yx}$.

$$49. f_x(x, y) = \frac{\partial}{\partial x} \left[\tan^{-1}(x^2 + y^3) \right] = \frac{2x}{1 + (x^2 + y^3)^2} \Rightarrow$$

$$f_{xy}(x, y) = \frac{\partial}{\partial y} \left\{ 2x \left[1 + (x^2 + y^3)^2 \right]^{-1} \right\} = 2x(-1) \left[1 + (x^2 + y^3)^2 \right]^{-2} (2)(x^2 + y^3)(3y^2) = -\frac{12xy^2(x^2 + y^3)}{\left[1 + (x^2 + y^3)^2 \right]^2}$$

$$\text{and } f_y(x, y) = \frac{\partial}{\partial y} \left[\tan^{-1}(x^2 + y^3) \right] = \frac{3y^2}{1 + (x^2 + y^3)^2} \Rightarrow$$

$$f_{yx}(x, y) = \frac{\partial}{\partial x} \left\{ 3y^2 \left[1 + (x^2 + y^3)^2 \right]^{-1} \right\} = 3y^2(-1) \left[1 + (x^2 + y^3)^2 \right]^{-2} (2)(x^2 + y^3)(2x) = -\frac{12xy^2(x^2 + y^3)}{\left[1 + (x^2 + y^3)^2 \right]^2},$$

so $f_{xy} = f_{yx}$.