14. 2

5. $\int_0^\pi \int_0^\pi \cos(x+y)\,dy\,dx = \int_0^\pi \left[\int_0^\pi \cos(x+y)\,dy\right] dx = \int_0^\pi \left[\sin(x+y)\right]_{y=0}^{y=\pi} dx = \int_0^\pi \left[\sin(x+\pi) - \sin x\right] dx$

$$= -2\int_0^\pi \sin x\,dx = 2\cos x\big|_0^\pi = -4$$

11. $\int_{-1}^1 \int_x^{2x} e^{x+y}\,dy\,dx = \int_{-1}^1 \left[\int_x^{2x} e^{x+y}\,dy\right] dx = \int_{-1}^1 \left[e^{x+y}\right]_{y=x}^{y=2x} dx = \int_{-1}^1 \left(e^{3x} - e^{2x}\right) dx = \left(\frac{1}{3}e^{3x} - \frac{1}{2}e^{2x}\right)\Big|_{-1}^1$

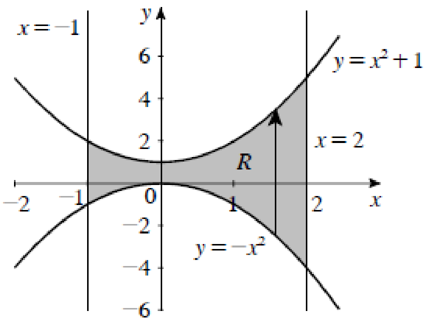
$$= \left(\tfrac{1}{3}e^3 - \tfrac{1}{2}e^2\right) - \left(\tfrac{1}{3}e^{-3} - \tfrac{1}{2}e^{-2}\right) = \frac{2e^6 - 3e^5 + 3e - 2}{6e^3}$$

20. $\iint_R xy\,dA = \int_{-1}^2 \int_{-x^2}^{1+x^2} xy\,dy\,dx$

$$= \int_{-1}^2 \left[\tfrac{1}{2}xy^2\right]_{y=-x^2}^{y=1+x^2} dx$$

$$= \tfrac{1}{2}\int_{-1}^2 \left(2x^3 + x\right) dx$$

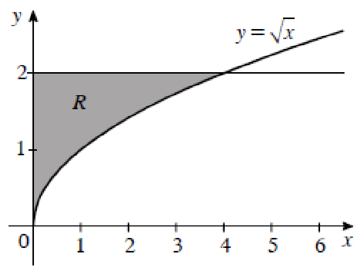
$$= \tfrac{1}{2}\left(\tfrac{1}{2}x^4 + \tfrac{1}{2}x^2\right)\Big|_{-1}^2 = \tfrac{9}{2}$$



57. $\int_0^4 \int_{\sqrt{x}}^2 \sin y^3\,dy\,dx = \int_0^2 \int_0^{y^2} \sin y^3\,dx\,dy = \int_0^2 \left[x\sin y^3\right]_{x=0}^{x=y^2} dy$

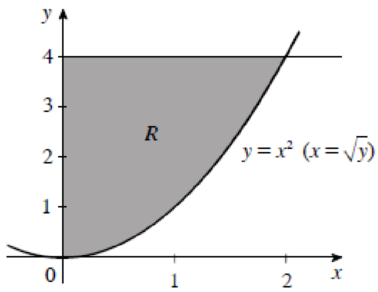
$$= \int_0^2 y^2 \sin y^3\,dy = -\tfrac{1}{3}\cos y^3\Big|_0^2$$

$$= \frac{1-\cos 8}{3}$$



58. $\int_0^2 \int_{x^2}^4 x\cos y^2\,dy\,dx = \int_0^4 \int_0^{\sqrt{y}} x\cos y^2\,dx\,dy = \int_0^4 \left[\tfrac{1}{2}x^2\cos y^2\right]_{x=0}^{x=\sqrt{y}} dy$

$$= \tfrac{1}{2}\int_0^4 y\cos y^2\,dy = \tfrac{1}{4}\sin y^2\Big|_0^4 = \tfrac{1}{4}\sin 16$$



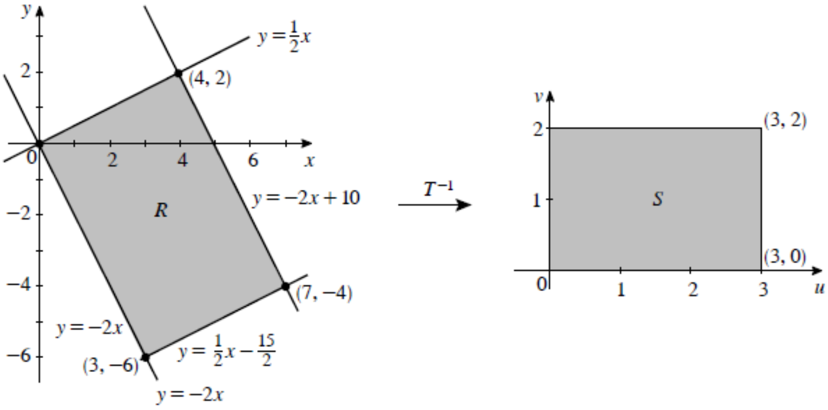
14. 8

13. To find T^{-1}, we solve the system of equations of $T: x = u + 2v,\ y = v - 2u$ for u and v, obtaining $T^{-1}: u = \frac{1}{5}(x - 2y),\ v = \frac{1}{5}(2x + y)$. Using this transformation, we obtain the region $S = T^{-1}(R)$. Next, we find

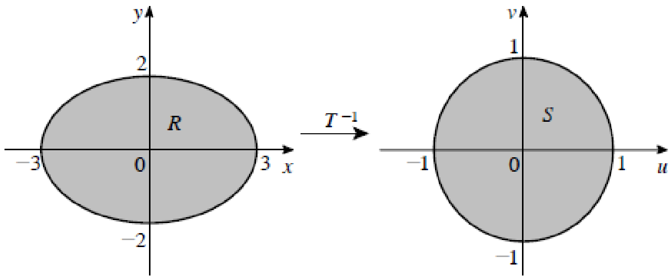
$$J = \frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 1 & 2 \\ -2 & 1 \end{vmatrix} = 5. \text{ Thus,}$$

$$\iint_R (x + y)\, dA = \iint_S [(u + 2v) + (v - 2u)] \left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\, dv = 5 \int_0^2 \int_0^3 (3v - u)\, du\, dv$$

$$= 5 \int_0^2 \left[3uv - \frac{1}{2}u^2\right]_{u=0}^{u=3} dv = 5 \int_0^2 \left(9v - \frac{9}{2}\right) dv = 5 \left(\frac{9}{2}v^2 - \frac{9}{2}v\right)\Big|_0^2 = 45$$



15. Here $T: x = 3u,\ y = 2v$. Then $4x^2 + 9y^2 = 36$ $\Rightarrow 4(3u)^2 + 9(2v)^2 = 36 \Leftrightarrow u^2 + v^2 = 1$, the circle with radius 1 centered at the origin of the uv-plane. Next, we find



$$J = \frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 3 & 0 \\ 0 & 2 \end{vmatrix} = 6, \text{ so}$$

$$\iint_R 2xy\, dA = 2 \iint_S (3u)(2v) \left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\, dv = 12 \cdot 6 \int_0^{2\pi} \int_0^1 (r\cos\theta)(r\sin\theta)\, r\, dr\, d\theta = 72 \int_0^{2\pi} \left[\frac{1}{4}r^4 \cos\theta \sin\theta\right]_{r=0}^{r=1} d\theta$$

$$= 72 \int_0^{2\pi} \frac{1}{4} \cos\theta \sin\theta\, d\theta = -9 \sin^2\theta\Big|_0^{2\pi} = 0$$

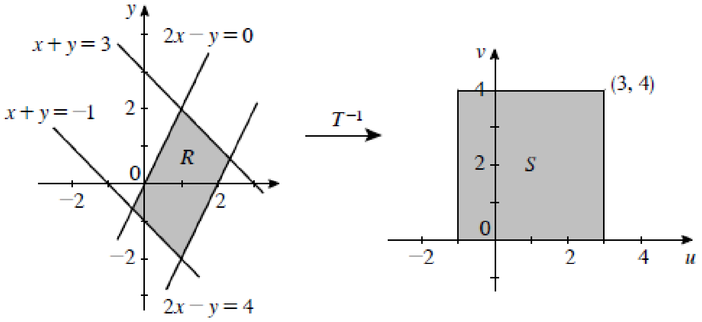
19. Using the result of Exercise 5, we see that T^{-1} maps the semicircular region R onto the quarter of the unit disk lying in the first quadrant of the uv-plane; that is, $T^{-1}(R) = S = \left\{(u, v) \mid u^2 + v^2 \le 1, u \ge 0, v \ge 0\right\}$. Next, we find

$$J = \frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 2u & -2v \\ 2v & 2u \end{vmatrix} = 4\left(u^2 + v^2\right), \text{ and so}$$

$$\iint_R \frac{1}{\sqrt{x^2 + y^2}}\, dA = \iint_S \frac{1}{\sqrt{(u^2 - v^2)^2 + (2uv)^2}} \left|\frac{\partial(x, y)}{\partial(u, v)}\right| du\, dv = \iint_S \frac{4\left(u^2 + v^2\right)}{\sqrt{(u^2 + v^2)^2}}\, du\, dv$$

$$= \iint_S 4\, du\, dv = 4\,(\text{area of } S) = 4\left(\frac{1}{4}\pi \cdot 1^2\right) = \pi$$

21. Let $u = x + y$ and $v = 2x - y$. Then $-1 \le u \le 3$ and $0 \le v \le 4$. Solving for x and y, we obtain $x = \frac{1}{3}(u + v)$ and $y = \frac{1}{3}(2u - v)$. Thus,



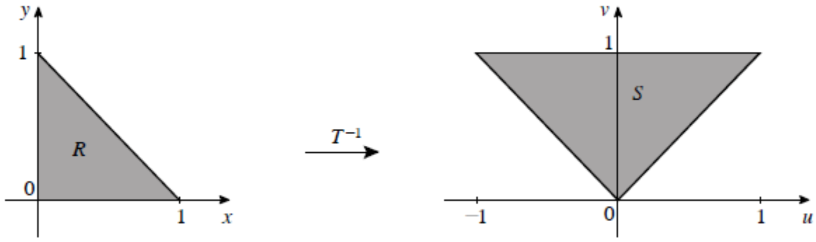
$$J = \frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{vmatrix} = -\frac{1}{3},$$

so

$$\iint_R (2x + y)\, dA = \int_{-1}^{3} \int_0^4 \left[\frac{2}{3}(u + v) + \frac{1}{3}(2u - v)\right] \left|-\frac{1}{3}\right| dv\, du = \frac{1}{9} \int_{-1}^{3} \int_0^4 (4u + v)\, dv\, du$$

$$= \frac{1}{9} \int_{-1}^{3} \left[4uv + \frac{1}{2}v^2\right]_{v=0}^{v=4} du = \frac{1}{9} \int_{-1}^{3} (16u + 8)\, du = \frac{8}{9}\left(u^2 + u\right)\Big|_{-1}^{3} = \frac{32}{3}$$

23. Let $T : u = x - y, v = x + y$. Then $T^{-1} : x = \frac{1}{2}(u + v), y = \frac{1}{2}(v - u)$. The triangular region R is mapped onto the triangular region S.



Here $J = \dfrac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{vmatrix} = \frac{1}{2}$, so

$$\iint_R \exp\left(\frac{x - y}{x + y}\right) dA = \int_0^1 \int_{-v}^{v} e^{u/v} \left|\frac{1}{2}\right| du\, dv = \frac{1}{2} \int_0^1 \left[v e^{u/v}\right]_{u=-v}^{u=v} dv = \frac{1}{2} \int_0^1 v\left(e - e^{-1}\right) dv = \frac{1}{2}\left(e - e^{-1}\right)\left(\frac{1}{2}v^2\right)\Big|_0^1$$

$$= \frac{e^2 - 1}{4e}$$