1.1 An Intuitive Introduction to Limits

7. a. True.

b. True. Since $\lim_{x\to 0^-} f(x) = 2$ and $\lim_{x\to 0^+} f(x) = 2$, $\lim_{x\to 0} f(x) = 2$.

c. False. Since $\lim_{x\to 2^-} f(x) = 2$ and $\lim_{x\to 2^+} f(x) = 2$, $\lim_{x\to 2} f(x) = 2 \neq 1$.

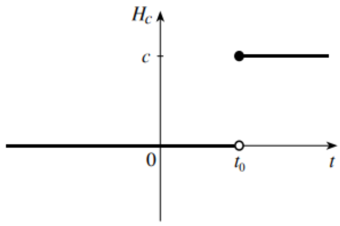
d. True.

e. True. $\lim_{x\to 4^+} f(x) = \infty$, which is another way of saying that $\lim_{x\to 4^+} f(x)$ does not exist.

f. False. From part e, we know that the right-hand limit does not exist. Therefore, $\lim_{x\to 4} f(x)$ does not exist.

25. $\lim_{x\to -1^+} [\![x]\!] = -1$

33. The graph of $H_c(t - t_0)$ is shown in the figure. If $c \neq 0$, then $\lim_{t\to t_0^-} H_c = 0$ and $\lim_{t\to t_0^+} H_c = c$. Since the right-hand limit is not equal to the left-hand limit, $\lim_{t\to t_0} H_c$ does not exist.



1.2 Techniques for Finding Limits

29.
$$\begin{aligned}\lim_{x\to -2}\left[xf(x) + \left(x^2+1\right)g(x)\right] &= \lim_{x\to -2}\left[xf(x)\right] + \lim_{x\to -2}\left[\left(x^2+1\right)g(x)\right] \\ &= \left(\lim_{x\to -2} x\right)\left[\lim_{x\to -2} f(x)\right] + \left[\lim_{x\to -2}\left(x^2+1\right)\right]\left[\lim_{x\to -2} g(x)\right] \\ &= (-2)\,2 + \left[\left(\lim_{x\to -2} x\right)^2 + \lim_{x\to -2} 1\right](3) = -4 + \left[(-2)^2+1\right](3) = 11\end{aligned}$$

46. $$\lim_{x\to 2}\frac{x^2 - x - 2}{x-2} = \lim_{x\to 2}\frac{(x-2)(x+1)}{x-2} = \lim_{x\to 2}(x+1) = 3$$

69. $$\lim_{x\to 0}\frac{\tan^2 x}{x} = \left(\lim_{x\to 0}\tan x\right)\left(\lim_{x\to 0}\frac{\tan x}{x}\right) = 0\cdot\left[\lim_{x\to 0}\left(\frac{\sin x}{\cos x}\cdot\frac{1}{x}\right)\right] = 0\cdot\left(\lim_{x\to 0}\frac{1}{\cos x}\right)\left(\lim_{x\to 0}\frac{\sin x}{x}\right) = 0\cdot 1\cdot 1 = 0$$

88. $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}\left(\sqrt{1-x}+2\right) = 2$ and $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}\left(1 + x^{3/2}\right) = 2$. Since $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = 2$, we conclude that $\lim_{x\to 1} f(x)$ exists and has a value of 2.

91. Let $g(x) = -x^2$ and $h(x) = x^2$ for all real x. Then $g(x) \leq f(x) \leq h(x)$ for all x. Since $\lim_{x\to 0} g(x) = \lim_{x\to 0} h(x) = 0$, the result follows from the Squeeze Theorem.