

3.  $f(t) = \sqrt{t}$  is defined for  $t \geq 0$ ,  $g(t) = \frac{1}{t-1}$  is not defined at 1, and  $h(t) = \ln t$  is defined for  $t > 0$ , so the domain of  $\mathbf{r}$  is  $(0, 1) \cup (1, \infty)$ .

6.  $f(t) = \sqrt[3]{t}$  is defined on  $(-\infty, \infty)$ ,  $g(t) = e^{1/t}$  is defined on  $(-\infty, 0) \cup (0, \infty)$ , and  $h(t) = \frac{1}{t+2}$  is defined on  $(-\infty, -2) \cup (-2, \infty)$ , so the domain of  $\mathbf{r}$  is  $(-\infty, -2) \cup (-2, 0) \cup (0, \infty)$ .

33.  $x + y + 2z = 1 \Rightarrow z = \frac{1}{2}(1 - x - y)$ , but  $x^2 + y^2 = 1$  can be parametrized by  $x = \cos t$ ,  $y = \sin t$ ,  $0 \leq t \leq 2\pi$ , so the curve is described by  $\mathbf{r}(t) = \left\langle \cos t, \sin t, \frac{1 - \cos t - \sin t}{2} \right\rangle$ ,  $0 \leq t \leq 2\pi$ .

$$39. \lim_{t \rightarrow 2} \left[ \sqrt{t} \mathbf{i} + \frac{t^2 - 4}{t - 2} \mathbf{j} + \frac{t}{t^2 + 1} \mathbf{k} \right] = \lim_{t \rightarrow 2} \sqrt{t} \mathbf{i} + \lim_{t \rightarrow 2} (t + 2) \mathbf{j} + \lim_{t \rightarrow 2} \frac{t}{t^2 + 1} \mathbf{k} = \sqrt{2} \mathbf{i} + 4 \mathbf{j} + \frac{2}{5} \mathbf{k}$$

$$41. \lim_{t \rightarrow \infty} \left\langle e^{-t}, \frac{1}{t}, \frac{2t^2}{t^2 + 1} \right\rangle = \left\langle \lim_{t \rightarrow \infty} e^{-t}, \lim_{t \rightarrow \infty} \frac{1}{t}, \lim_{t \rightarrow \infty} \frac{2}{1 + (1/t^2)} \right\rangle = \langle 0, 0, 2 \rangle$$

$$42. \lim_{t \rightarrow -\infty} \left[ \frac{t-1}{2t+1} \mathbf{i} + e^{2t} \mathbf{j} + \tan^{-1} t \mathbf{k} \right] = \lim_{t \rightarrow -\infty} \frac{1 - \frac{1}{t}}{2 + \frac{1}{t}} \mathbf{i} + \lim_{t \rightarrow -\infty} e^{2t} \mathbf{j} + \lim_{t \rightarrow -\infty} \tan^{-1} t \mathbf{k} = \frac{1}{2} \mathbf{i} - \frac{\pi}{2} \mathbf{k}$$

45. Since  $f(t) = \frac{\cos t - 1}{t}$  has domain  $(-\infty, 0) \cup (0, \infty)$ ,  $g(t) = \frac{\sqrt{t}}{1 + 2t}$  is continuous on  $[0, \infty)$ , and  $h(t) = te^{-1/t}$  is continuous on  $(-\infty, 0)$  and  $(0, \infty)$ , we see that  $\mathbf{r}$  is continuous on  $(0, \infty)$ .

47. Since  $f(t) = e^{-t}$  is continuous on  $(-\infty, \infty)$ ,  $g(t) = \cos \sqrt{4-t}$  is continuous on  $(-\infty, 4]$ , and  $h(t) = 1/(t^2 - 1)$  is continuous on  $(-\infty, -1)$ ,  $(-1, 1)$ , and  $(1, \infty)$ , we see that  $\mathbf{r}$  is continuous on  $(-\infty, -1)$ ,  $(-1, 1)$ , and  $(1, 4]$ .

## 12.2 Differentiation and Integration of Vector-Valued Functions ET 11.2

$$4. \mathbf{r}(t) = \langle t \cos t, t \sin t, \tan t \rangle \Rightarrow \mathbf{r}'(t) = \langle \cos t - t \sin t, \sin t + t \cos t, \sec^2 t \rangle \Rightarrow \\ \mathbf{r}''(t) = \langle -t \cos t - 2 \sin t, 2 \cos t - t \sin t, 2 \sec^2 t \tan t \rangle$$

$$6. \mathbf{r}(t) = e^{-t} \mathbf{i} + te^t \mathbf{j} + e^{-2t} \mathbf{k} \Rightarrow \mathbf{r}'(t) = -e^{-t} \mathbf{i} + (t+1)e^t \mathbf{j} - 2e^{-2t} \mathbf{k} \Rightarrow \mathbf{r}''(t) = e^{-t} \mathbf{i} + (t+2)e^t \mathbf{j} + 4e^{-2t} \mathbf{k}$$

$$7. \mathbf{r}(t) = e^{-t} \sin t \mathbf{i} + e^{-t} \cos t \mathbf{j} + \tan^{-1} t \mathbf{k} \Rightarrow \mathbf{r}'(t) = (\cos t - \sin t)e^{-t} \mathbf{i} - (\cos t + \sin t)e^{-t} \mathbf{j} + \frac{1}{t^2 + 1} \mathbf{k} \Rightarrow \\ \mathbf{r}''(t) = -2e^{-t} \cos t \mathbf{i} + 2e^{-t} \sin t \mathbf{j} - \frac{2t}{(t^2 + 1)^2} \mathbf{k}$$

$$8. \mathbf{r}(t) = \langle \sin^{-1} t, \sec t, \ln |t| \rangle \Rightarrow \mathbf{r}'(t) = \left\langle \frac{1}{\sqrt{1-t^2}}, \sec t \tan t, \frac{1}{t} \right\rangle \Rightarrow \mathbf{r}''(t) = \left\langle \frac{t}{(1-t^2)^{3/2}}, \sec t (\sec^2 t + \tan^2 t), -\frac{1}{t^2} \right\rangle$$

$$17. \mathbf{r}(t) = t\mathbf{i} + 2t\mathbf{j} + 3t\mathbf{k} \Rightarrow \mathbf{r}'(t) = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \Rightarrow \mathbf{r}'(1) = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}, \text{ so } |\mathbf{r}'(1)| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}, \text{ and therefore}$$

$$\mathbf{T}(1) = \frac{\mathbf{r}'(1)}{|\mathbf{r}'(1)|} = \frac{\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}}{\sqrt{14}} = \frac{\sqrt{14}}{14}\mathbf{i} + \frac{\sqrt{14}}{7}\mathbf{j} + \frac{3\sqrt{14}}{14}\mathbf{k}.$$

$$19. \mathbf{r}(t) = 2 \sin 2t \mathbf{i} + 3 \cos 2t \mathbf{j} + 3\mathbf{k} \Rightarrow \mathbf{r}'(t) = 4 \cos 2t \mathbf{i} - 6 \sin 2t \mathbf{j} \Rightarrow \mathbf{r}'\left(\frac{\pi}{6}\right) = 2\mathbf{i} - 3\sqrt{3}\mathbf{j}, \text{ so}$$

$$|\mathbf{r}'\left(\frac{\pi}{6}\right)| = \sqrt{2^2 + (-3\sqrt{3})^2} = \sqrt{31}, \text{ and therefore } \mathbf{T}\left(\frac{\pi}{6}\right) = \frac{\mathbf{r}'\left(\frac{\pi}{6}\right)}{|\mathbf{r}'\left(\frac{\pi}{6}\right)|} = \frac{1}{\sqrt{31}}(2\mathbf{i} - 3\sqrt{3}\mathbf{j}) = \frac{2\sqrt{31}}{31}\mathbf{i} - \frac{3\sqrt{93}}{31}\mathbf{j}.$$

$$26. \mathbf{r}(t) = e^{-t} \cos t \mathbf{i} + e^{-t} \sin t \mathbf{j} + \sin^{-1} t \mathbf{k} \Rightarrow \mathbf{r}'(t) = -e^{-t}(\sin t + \cos t)\mathbf{i} - e^{-t}(\sin t - \cos t)\mathbf{j} + \frac{1}{\sqrt{1-t^2}}\mathbf{k}, \text{ so a vector}$$

equation of the tangent line when  $t = 0$  is  $\mathbf{r}'(t) = \mathbf{r}(0) + t\mathbf{r}'(0) = \mathbf{i} + t(-\mathbf{i} + \mathbf{j} + \mathbf{k}) = (1-t)\mathbf{i} + t\mathbf{j} + t\mathbf{k}$  and parametric equations are  $x = 1 - t, y = t, z = t$ .

$$30. \int_1^2 \left[ \sqrt{t-1}\mathbf{i} + \frac{1}{\sqrt{t}}\mathbf{j} + (2t-1)^5\mathbf{k} \right] dt = \left[ \frac{2}{3}(t-1)^{3/2}\mathbf{i} + 2t^{1/2}\mathbf{j} + \frac{1}{12}(2t-1)^6\mathbf{k} \right]_1^2 = \left( \frac{2}{3}\mathbf{i} + 2\sqrt{2}\mathbf{j} + \frac{243}{4}\mathbf{k} \right) - \left( 2\mathbf{j} + \frac{1}{12}\mathbf{k} \right) = \frac{2}{3}\mathbf{i} + 2(\sqrt{2}-1)\mathbf{j} + \frac{182}{3}\mathbf{k}$$

$$31. \int (\sin 2t \mathbf{i} + \cos 2t \mathbf{j} + e^{-t} \mathbf{k}) dt = -\frac{1}{2} \cos 2t \mathbf{i} + \frac{1}{2} \sin 2t \mathbf{j} - e^{-t} \mathbf{k} + \mathbf{C}$$

$$32. \int (te^t \mathbf{i} + 2\mathbf{j} - \sec^2 t \mathbf{k}) dt = (t-1)e^t \mathbf{i} + 2t\mathbf{j} - \tan t \mathbf{k} + \mathbf{C}$$

$$33. \int (t \cos t \mathbf{i} + t \sin t^2 \mathbf{j} - te^{t^2} \mathbf{k}) dt = (\cos t + t \sin t)\mathbf{i} - \frac{1}{2} \cos t^2 \mathbf{j} - \frac{1}{2} e^{t^2} + \mathbf{C}. \text{ (Note that we have integrated } \int t \cos t dt \text{ by parts.)}$$

$$34. \int \left( \frac{1}{1+t^2} \mathbf{i} + \frac{t}{1+2t^2} \mathbf{j} - \frac{1}{\sqrt{1-t^2}} \mathbf{k} \right) dt = \tan^{-1} t \mathbf{i} + \frac{1}{4} \ln(2t^2+1) \mathbf{j} - \sin^{-1} t \mathbf{k} + \mathbf{C}$$

## 13.1 Functions of Two or More Variables

ET 12.1

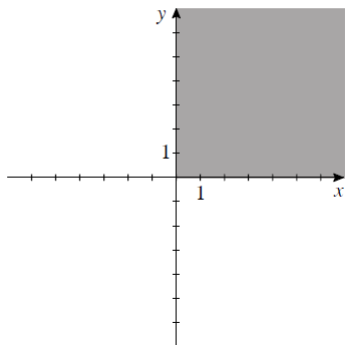
5. Since  $f(x, y) = x + 3y - 1$  is defined for all pairs  $(x, y)$  of real numbers, the domain of  $f$  is  $\{(x, y) \mid -\infty < x < \infty, -\infty < y < \infty\}$ . The range of  $f$  is  $\{z \mid -\infty < z < \infty\}$ .

7. Since  $u \neq v$ , the domain of  $f$  is  $\{(u, v) \mid u \neq v\}$ . The range of  $f$  is  $\{z \mid -\infty < z < \infty\}$ .

9. We require that the radicand  $4 - x^2 - y^2 \geq 0 \Leftrightarrow x^2 + y^2 \leq 4$ , so the domain of  $g$  is  $\{(x, y) \mid x^2 + y^2 \leq 4\}$ . The range of  $g$  is  $\{z \mid 0 \leq z \leq 2\}$ .

15. The domain of  $f$  is

$$\{(x, y) \mid x \geq 0 \text{ and } y \geq 0\}$$



17. The domain of  $f$  is

$$\{(u, v) \mid u \neq v, u \neq -v\}$$

