12.1 Vector-Valued Functions ET 11.1

37. $\lim_{t\to 0}\left[\left(t^2+1\right)\mathbf{i}+\cos t\,\mathbf{j}-3\mathbf{k}\right]=\mathbf{i}+\mathbf{j}-3\mathbf{k}$

38. $\lim_{t\to 0}\left\langle e^{-t},\frac{\sin t}{t},\cos t\right\rangle=\left\langle \lim_{t\to 0}e^{-t},\lim_{t\to 0}\frac{\sin t}{t},\lim_{t\to 0}\cos t\right\rangle=\langle 1,1,1\rangle$

45. Since $f(t)=\dfrac{\cos t-1}{t}$ has domain $(-\infty,0)\cup(0,\infty)$, $g(t)=\dfrac{\sqrt{t}}{1+2t}$ is continuous on $[0,\infty)$, and $h(t)=te^{-1/t}$ is continuous on $(-\infty,0)$ and $(0,\infty)$, we see that $\mathbf{r}$ is continuous on $(0,\infty)$.

47. Since $f(t)=e^{-t}$ is continuous on $(-\infty,\infty)$, $g(t)=\cos\sqrt{4-t}$ is continuous on $(-\infty,4]$, and $h(t)=1/\left(t^2-1\right)$ is continuous on $(-\infty,-1)$, $(-1,1)$, and $(1,\infty)$, we see that $\mathbf{r}$ is continuous on $(-\infty,-1)$, $(-1,1)$, and $(1,4]$.

12.2 Differentiation and Integration of Vector-Valued Functions ET 11.2

3. $\mathbf{r}(t)=\left\langle t^2-1,\sqrt{t^2+1}\right\rangle\Rightarrow\mathbf{r}'(t)=\left\langle 2t,\frac{t}{\sqrt{t^2+1}}\right\rangle$ and since

$$\frac{d}{dt}\frac{t}{\sqrt{t^2+1}}=\frac{d}{dt}\left[t\left(t^2+1\right)^{-1/2}\right]=\left(t^2+1\right)^{-1/2}+t\left(-\tfrac{1}{2}\right)\left(t^2+1\right)^{-3/2}(2t)=\frac{1}{\left(t^2+1\right)^{3/2}},$$

$\mathbf{r}''(t)=\left\langle 2,\frac{1}{\left(t^2+1\right)^{3/2}}\right\rangle.$

27. $\int\left(t\mathbf{i}+2t^2\mathbf{j}+3\mathbf{k}\right)dt=\frac{1}{2}t^2\mathbf{i}+\frac{2}{3}t^3\mathbf{j}+3t\mathbf{k}+\mathbf{C}$

37. $\mathbf{r}(t)=\int\mathbf{r}'(t)\,dt=\int\left(2e^{2t}\mathbf{i}+3e^{-t}\mathbf{j}+e^t\mathbf{k}\right)dt=e^{2t}\mathbf{i}-3e^{-t}\mathbf{j}+e^t\mathbf{k}+\mathbf{C}$ and $\mathbf{r}(0)=\mathbf{i}-\mathbf{j}+\mathbf{k}\Rightarrow$ $\mathbf{i}-3\mathbf{j}+\mathbf{k}+\mathbf{C}=\mathbf{i}-\mathbf{j}+\mathbf{k}\Rightarrow\mathbf{C}=2\mathbf{j}$, so $\mathbf{r}(t)=e^{2t}\mathbf{i}-\left(3e^{-t}-2\right)\mathbf{j}+e^t\mathbf{k}$.

53. $\frac{d}{dt}\left[\mathbf{r}(t)\times\mathbf{r}'(t)\right]=\mathbf{r}'(t)\times\mathbf{r}'(t)+\mathbf{r}(t)\times\mathbf{r}''(t)=\mathbf{r}(t)\times\mathbf{r}''(t)$ since $\mathbf{r}'(t)\times\mathbf{r}'(t)=\mathbf{0}$.