Not homework. Just supplement.

11.3 The Dot Product — ET 10.3

42. $\mathbf{a} = \langle 1, 2, 0 \rangle$ and $\mathbf{b} = \langle -3, 0, -4 \rangle$.

a. $\text{proj}_{\mathbf{a}}\,\mathbf{b} = \left(\dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\right)\mathbf{a} = \left(\dfrac{\langle 1,2,0\rangle\cdot\langle -3,0,-4\rangle}{1+4}\right)\langle 1,2,0\rangle = -\frac{3}{5}\langle 1,2,0\rangle = \left\langle -\frac{3}{5}, -\frac{6}{5}, 0\right\rangle$

b. $\text{proj}_{\mathbf{b}}\,\mathbf{a} = \left(\dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{b}|^2}\right)\mathbf{b} = \left(\dfrac{\langle 1,2,0\rangle\cdot\langle -3,0,-4\rangle}{9+16}\right)\langle -3,0,-4\rangle = -\frac{3}{25}\langle -3,0,-4\rangle = \left\langle \frac{9}{25}, 0, \frac{12}{25}\right\rangle$

47. $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$. Because
$\text{proj}_{\mathbf{a}}\,\mathbf{b} = \left(\dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\right)\mathbf{a} = \left[\dfrac{(\mathbf{i}+2\mathbf{j}+3\mathbf{k})\cdot(2\mathbf{i}-\mathbf{j}+\mathbf{k})}{1+4+9}\right](\mathbf{i}+2\mathbf{j}+3\mathbf{k}) = \left(\dfrac{2-2+3}{14}\right)(\mathbf{i}+2\mathbf{j}+3\mathbf{k}) = \frac{3}{14}\mathbf{i} + \frac{3}{7}\mathbf{j} + \frac{9}{14}\mathbf{k}$ is parallel to $\mathbf{a}$, we can write

$$\mathbf{b} = \text{proj}_{\mathbf{a}}\,\mathbf{b} + (\mathbf{b} - \text{proj}_{\mathbf{a}}\,\mathbf{b}) = \left(\tfrac{3}{14}\mathbf{i} + \tfrac{3}{7}\mathbf{j} + \tfrac{9}{14}\mathbf{k}\right) + \left[(2\mathbf{i}-\mathbf{j}+\mathbf{k}) - \left(\tfrac{3}{14}\mathbf{i} + \tfrac{3}{7}\mathbf{j} + \tfrac{9}{14}\mathbf{k}\right)\right]$$
$$= \left(\tfrac{3}{14}\mathbf{i} + \tfrac{3}{7}\mathbf{j} + \tfrac{9}{14}\mathbf{k}\right) + \left(\tfrac{25}{14}\mathbf{i} - \tfrac{10}{7}\mathbf{j} + \tfrac{5}{14}\mathbf{k}\right)$$

11.4 The Cross Product — ET 10.4

13. One such vector is $\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -3 & 4 \\ -1 & 1 & -2 \end{vmatrix} = 2\mathbf{i} - \mathbf{k}$; another is $\mathbf{b}\times\mathbf{a} = -2\mathbf{i} + \mathbf{k}$.

17. For $P(1,0,0)$, $Q(0,1,0)$, and $R(0,0,1)$, $\overrightarrow{PQ} = -\mathbf{i} + \mathbf{j}$ and $\overrightarrow{PR} = -\mathbf{i} + \mathbf{k}$, so $\overrightarrow{PQ}\times\overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, so the area of $\triangle PQR$ is $\frac{1}{2}\left|\overrightarrow{PQ}\times\overrightarrow{PR}\right| = \frac{1}{2}|\mathbf{i}+\mathbf{j}+\mathbf{k}| = \frac{1}{2}\sqrt{1+1+1} = \frac{\sqrt{3}}{2}$.

35. For $P(1,0,1)$, $Q(2,3,1)$, $R(-1,2,-3)$, and $S\left(\frac{2}{3},-1,1\right)$, $\overrightarrow{PQ} = \langle 1,3,0\rangle$, $\overrightarrow{PR} = \langle -2,2,-4\rangle$, and $\overrightarrow{PS} = \left\langle -\frac{1}{3}, -1, 0\right\rangle$, so $\overrightarrow{PQ}\cdot\left(\overrightarrow{PR}\times\overrightarrow{PS}\right) = \begin{vmatrix} 1 & 3 & 0 \\ -2 & 2 & -4 \\ -\frac{1}{3} & -1 & 0 \end{vmatrix} = 1(0-4) - 3\left(0 - \frac{4}{3}\right) = 0$. Thus, the points are coplanar.

11.5 Lines and Planes in Space — ET 10.5

12. The direction of the given line is the same as that of $\mathbf{v} = \langle 3,-3,1\rangle$, so parametric equations of the required line are $x = -1 + 3t$, $y = 3 - 3t$, $z = -2 + t$. To find the point where the line intersects the yz-plane, set $x = 0 \Rightarrow t = \frac{1}{3} \Rightarrow y = 2$ and $z = -\frac{5}{3}$. Thus, the required point is $\left(0, 2, -\frac{5}{3}\right)$.

27. A normal to the given plane is $\mathbf{n} = \langle 2,3,-1\rangle$. Because the required plane is parallel to the given plane, $\mathbf{n}$ is also normal to the required plane, so an equation is $2(x-3) + 3(y-6) - 1(z+2) = 0 \Leftrightarrow 2x + 3y - z = 26$.

43. A normal to the plane $x + y + 2z = 6$ is $\mathbf{n} = \langle 1,1,2\rangle$ and a vector parallel to the line $L: x = 1 + t, y = 2 + t, z = -1 + t$ is $\mathbf{v} = \langle 1,1,1\rangle$, so the angle between the normal to the plane and the line is $\theta = \cos^{-1}\left(\dfrac{|\mathbf{n}\cdot\mathbf{v}|}{|\mathbf{n}|\,|\mathbf{v}|}\right) = \cos^{-1}\left(\dfrac{|\langle 1,1,2\rangle\cdot\langle 1,1,1\rangle|}{\sqrt{1+1+4}\sqrt{1+1+1}}\right) = \cos^{-1}\dfrac{4}{\sqrt{6}\sqrt{3}} \approx 19.5^\circ$. Therefore, the required angle is about $90^\circ - 19.5^\circ = 70.5^\circ$.