2.6

31. $f(x) = \sin 2x + \tan\sqrt{x} \Rightarrow f'(x) = (\cos 2x)\,2 + \left(\sec^2\sqrt{x}\right)\left(\frac{1}{2}x^{-1/2}\right) = 2\cos 2x + \dfrac{\sec^2\sqrt{x}}{2\sqrt{x}}$

47. $y = \sin^2\left(\dfrac{1+x}{1-x}\right) \Rightarrow$

$$\frac{dy}{dx} = 2\sin\left(\frac{1+x}{1-x}\right)\cos\left(\frac{1+x}{1-x}\right)\frac{d}{dx}\left(\frac{1+x}{1-x}\right) = 2\sin\left(\frac{1+x}{1-x}\right)\cos\left(\frac{1+x}{1-x}\right)\cdot\frac{(1-x)(1)-(1+x)(-1)}{(1-x)^2}$$
$$= \frac{4}{(1-x)^2}\sin\left(\frac{1+x}{1-x}\right)\cos\left(\frac{1+x}{1-x}\right)$$

52. $y = x\tan^2(2x+3) \quad\Rightarrow$

$$\frac{dy}{dx} = x\frac{d}{dx}\tan^2(2x+3) + \tan^2(2x+3)\frac{d}{dx}(x) = x\left[2\tan(2x+3)\right]\frac{d}{dx}\left[\tan(2x+3)\right] + \tan^2(2x+3)$$
$$= 2x\tan(2x+3)\sec^2(2x+3)\frac{d}{dx}(2x+3) + \tan^2(2x+3) = \tan(2x+3)\left[4x\sec^2(2x+3) + \tan(2x+3)\right]$$

74. $F'(x) = \frac{d}{dx}\left\{f(x^a) + [f(x)]^a\right\} = \frac{d}{dx}f(x^a) + \frac{d}{dx}[f(x)]^a = f'(x^a)\frac{d}{dx}(x^a) + a[f(x)]^{a-1}\frac{d}{dx}f(x)$

$$= ax^{a-1}f'(x^a) + af'(x)[f(x)]^{a-1}$$

99. $f(x) = \begin{cases} x^2\sin\frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

Since

$$\frac{d}{dx}\left(x^2\sin\frac{1}{x}\right) = x^2\frac{d}{dx}\left(\sin\frac{1}{x}\right) + \sin\frac{1}{x}\frac{d}{dx}\left(x^2\right) = x^2\cos\frac{1}{x}\frac{d}{dx}\left(\frac{1}{x}\right) + 2x\sin\frac{1}{x} = x^2\left(-\frac{1}{x^2}\right)\cos\frac{1}{x} + 2x\sin\frac{1}{x}$$
$$= -\cos\frac{1}{x} + 2x\sin\frac{1}{x}$$

and $\lim\limits_{h\to 0}\dfrac{f(h)-f(0)}{h} = \lim\limits_{h\to 0}\dfrac{h^2\sin\frac{1}{h}}{h} = \lim\limits_{h\to 0} h\sin\dfrac{1}{h} = 0$ (by the Squeeze Theorem), we see that

$$f'(x) = \begin{cases} 2x\sin\frac{1}{x} - \cos\frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Next,

$$\frac{d}{dx}\left(2x\sin\frac{1}{x} - \cos\frac{1}{x}\right) = 2x\frac{d}{dx}\left(\sin\frac{1}{x}\right) + \sin\frac{1}{x}\frac{d}{dx}(2x) + \sin\frac{1}{x}\frac{d}{dx}\left(\frac{1}{x}\right) = 2x\left(-\frac{1}{x^2}\right)\cos\frac{1}{x} + 2\sin\frac{1}{x} + \left(-\frac{1}{x^2}\right)\sin\frac{1}{x}$$
$$= 2\sin\frac{1}{x} - \frac{2}{x}\cos\frac{1}{x} - \frac{1}{x^2}\sin\frac{1}{x}$$

Thus, $f''(x) = \left(2 - \frac{1}{x^2}\right)\sin\frac{1}{x} - \frac{2}{x}\cos\frac{1}{x}$ for $x \neq 0$. Since $\frac{d}{dx}\sin\frac{1}{x}$ does not exist at $x = 0$, we conclude that $f''(x)$ does not exist at $x = 0$.

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9. $\dfrac{xy}{x^2+y^2} = x+1 \;\Rightarrow\; \dfrac{\left(x^2+y^2\right)\dfrac{d}{dx}(xy) - xy\dfrac{d}{dx}\left(x^2+y^2\right)}{\left(x^2+y^2\right)^2} = 1 \;\Rightarrow$

$\dfrac{\left(x^2+y^2\right)(y+xy') - xy\dfrac{d}{dx}\left(x^2+y^2\right)}{\left(x^2+y^2\right)^2} = 1 \Rightarrow \dfrac{\left(x^2+y^2\right)(y+xy') - xy\,(2x+2yy')}{\left(x^2+y^2\right)^2} = 1$

$\Rightarrow x^2y + x^3y' + y^3 + xy^2y' - 2x^2y - 2xy^2y' = \left(x^2+y^2\right)^2 = x^4 + 2x^2y^2 + y^4 \Rightarrow$

$\left(x^3 - xy^2\right)y' = x^4 + 2x^2y^2 + y^4 - y^3 + x^2y \Rightarrow y' = \dfrac{x^4+2x^2y^2+x^2y-y^3+y^4}{x\left(x^2-y^2\right)}$

Alternate Solution: Start by multiplying both sides of the original equation by x^2+y^2:

$xy = \left(x^2+y^2\right)(x+1) = x^3+x^2+xy^2+y^2 \Rightarrow y + xy' = 3x^2+2x+y^2+2xyy'+2yy' \Rightarrow y' = \dfrac{3x^2+2x+y^2-y}{x-2xy-2y}$

17. $\tan^2\left(x^3+y^3\right) = xy \Rightarrow \left[2\tan\left(x^3+y^3\right)\sec^2\left(x^3+y^3\right)\right]\left(3x^2+3y^2y'\right) = y + xy' \Rightarrow$

$\left[6y^2\tan\left(x^3+y^3\right)\sec^2\left(x^3+y^3\right) - x\right]y' = y - 6x^2\tan\left(x^3+y^3\right)\sec^2\left(x^3+y^3\right) \Rightarrow$

$$y' = \frac{y - 6x^2\tan\left(x^3+y^3\right)\sec^2\left(x^3+y^3\right)}{6y^2\tan\left(x^3+y^3\right)\sec^2\left(x^3+y^3\right) - x}$$

25. $xy^2 - x^2y - 2 = 0 \Rightarrow y^2 + 2xyy' - 2xy - x^2y' = 0$. At $(1,-1)$, $1 - 2y' + 2 - y' = 0 \Rightarrow y' = 1$.

31. $\sin x + \cos y = 1 \Rightarrow \cos x - (\sin y)\,y' = 0 \Rightarrow y' = \frac{\cos x}{\sin y}$. Differentiating both sides of the next-to-last expression yields $-\sin x - (\cos y)\left(y'\right)^2 - (\sin y)\,y'' = 0 \Rightarrow$

$$y'' = -\frac{\sin x + (\cos y)\left(y'\right)^2}{\sin y} = -\frac{\sin x + \cos y\left(\frac{\cos x}{\sin y}\right)^2}{\sin y} = -\frac{\sin x\sin^2 y + \cos y\cos^2 x}{\sin^3 y}$$

35. $y^2 - xy^2 - x^3 = 0 \Rightarrow 2yy' - y^2 - 2xyy' - 3x^2 = 0$. At $\left(\frac{1}{2},\frac{1}{2}\right)$, $2\left(\frac{1}{2}\right)y' - \left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)y' - 3\left(\frac{1}{2}\right)^2 = 0 \Rightarrow$ $y' = 2$, so an equation of the tangent line is $y - \frac{1}{2} = 2\left(x - \frac{1}{2}\right)$ or $y = 2x - \frac{1}{2}$.