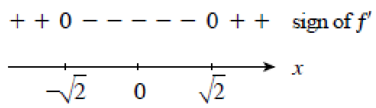
3. 3

11. $f(x) = x^3 - 6x + 1 \Rightarrow f'(x) = 3x^2 - 6 = 3\left(x^2 - 2\right) = 3\left(x + \sqrt{2}\right)\left(x - \sqrt{2}\right)$ is continuous everywhere and has zeros at $-\sqrt{2}$ and $\sqrt{2}$, the critical numbers of f. The sign diagram of f' is shown.

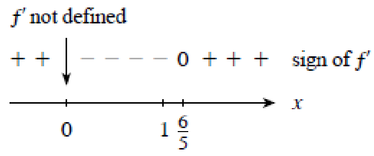


a. f is increasing on $\left(-\infty, -\sqrt{2}\right)$ and $\left(\sqrt{2}, \infty\right)$ and decreasing on $\left(-\sqrt{2}, \sqrt{2}\right)$.

b. f has a relative maximum of $f\left(-\sqrt{2}\right) = 1 + 4\sqrt{2}$ and a relative minimum of $f\left(\sqrt{2}\right) = 1 - 4\sqrt{2}$.

27. $f(x) = x^{2/3}(x - 3) \Rightarrow$

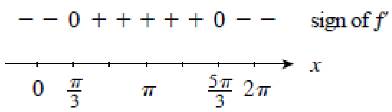
$f'(x) = \frac{2}{3}x^{-1/3}(x - 3) + x^{2/3} = \frac{1}{3}x^{-1/3}[2(x - 3) + 3x] = \dfrac{5x - 6}{3x^{1/3}}$ is discontinuous at 0 and has a zero at $\frac{6}{5}$, so 0 and $\frac{6}{5}$ are critical numbers of f. The sign diagram of f' is shown.



a. f is increasing on $(-\infty, 0)$ and $\left(\frac{6}{5}, \infty\right)$ and decreasing on $\left(0, \frac{6}{5}\right)$.

b. f has a relative maximum of $f(0) = 0$ and a relative minimum of $f\left(\frac{6}{5}\right) \approx -2.03$.

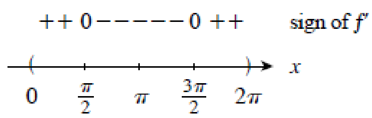
31. $f(x) = x - 2\sin x$, $0 < x < 2\pi \Rightarrow f'(x) = 1 - 2\cos x$ is continuous on $(0, 2\pi)$ and has zeros where $1 - 2\cos x = 0 \Leftrightarrow \cos x = \frac{1}{2} \Rightarrow x = \frac{\pi}{3}$ or $\frac{5\pi}{3}$. The sign diagram of f' is shown.



a. f is decreasing on $\left(0, \frac{\pi}{3}\right)$ and $\left(\frac{5\pi}{3}, 2\pi\right)$ and increasing on $\left(\frac{\pi}{3}, \frac{5\pi}{3}\right)$.

b. f has a relative minimum of $f\left(\frac{\pi}{3}\right) \approx -0.68$ and a relative maximum of $f\left(\frac{5\pi}{3}\right) \approx 6.97$.

35. $f(x) = x\sin x + \cos x$, $0 < x < 2\pi \Rightarrow f'(x) = \sin x + x\cos x - \sin x = x\cos x$ is continuous everywhere and has zeros where $x\cos x = 0 \Rightarrow x = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ in $(0, 2\pi)$. The sign diagram of f' is shown.



a. f is increasing on $\left(0, \frac{\pi}{2}\right)$ and $\left(\frac{3\pi}{2}, 2\pi\right)$ and decreasing on $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$.

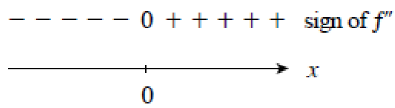
b. f has a relative maximum of $f\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$ and a relative minimum of $f\left(\frac{3\pi}{2}\right) = -\frac{3\pi}{2}$.

54. $f(x) = \tan x - x \Rightarrow f'(x) = \sec^2 x - 1 > 0$ for x in $\left(0, \frac{\pi}{2}\right) \Rightarrow f$ is increasing on $\left(0, \frac{\pi}{2}\right)$. Since $f(0) = 0$, we have $f(x) > 0$ for x in $\left(0, \frac{\pi}{2}\right)$, and so $\tan x - x > 0 \Rightarrow \tan x > x$ for x in $\left(0, \frac{\pi}{2}\right)$.

3.4

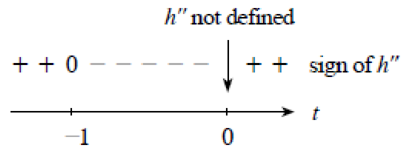
11. $f(x) = x^3 - 6x \Rightarrow f'(x) = 3x^2 - 6 \Rightarrow f''(x) = 6x = 0 \Leftrightarrow x = 0$.

The sign diagram of f'' is shown at right. We see that f is concave downward on $(-\infty, 0)$ and concave upward on $(0, \infty)$. It has an inflection point at $(0, 0)$.



17. $h(t) = \frac{1}{3}t^2 + \frac{3}{5}t^{5/3} \Rightarrow h'(t) = \frac{2}{3}t + t^{2/3} \Rightarrow h''(t) = \frac{2}{3} + \frac{2}{3}t^{-1/3} = \frac{2}{3}\left(1 + t^{-1/3}\right) = \dfrac{2\left(t^{1/3} + 1\right)}{3t^{1/3}}$.

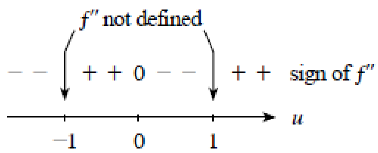
The sign diagram of h'' is shown at right. We see that h is concave upward on $(-\infty, -1)$ and $(0, \infty)$ and concave downward on $(-1, 0)$. It has inflection points at $\left(-1, -\frac{4}{15}\right)$ and $(0, 0)$.



23. $f(u) = \dfrac{u}{u^2 - 1} \Rightarrow f'(u) = \dfrac{\left(u^2 - 1\right)(1) - u(2u)}{\left(u^2 - 1\right)^2} = -\dfrac{u^2 + 1}{\left(u^2 - 1\right)^2} \Rightarrow$

$$f''(u) = \frac{-\left(u^2 - 1\right)^2(2u) + \left(u^2 + 1\right)(2)\left(u^2 - 1\right)(2u)}{\left(u^2 - 1\right)^4} = \frac{(2u)\left(u^2 - 1\right)\left(-u^2 + 1 + 2u^2 + 2\right)}{\left(u^2 - 1\right)^4} = \frac{2u\left(u^2 + 3\right)}{\left(u^2 - 1\right)^3}.$$

The sign diagram of f'' is shown at right. We see that f is concave downward on $(-\infty, -1)$ and $(0, 1)$ and concave upward on $(-1, 0)$ and $(1, \infty)$. It has an inflection point at $(0, 0)$.



47. $f(x) = 2\sin x + \sin 2x,\ 0 < x < \pi \Rightarrow$

$f'(x) = 2\cos x + 2\cos 2x = 2\cos x + 2\left(2\cos^2 x - 1\right) = 2\left(2\cos^2 x + \cos x - 1\right) = 2(2\cos x - 1)(\cos x + 1) = 0 \Rightarrow$

$x = \frac{\pi}{3}$, the only critical number in $(0, \pi)$. $f''(x) = -2\sin x - 4\sin 2x$ and $f''\left(\frac{\pi}{3}\right) = -3\sqrt{3} < 0$, so by the SDT, f has a relative maximum of $f\left(\frac{\pi}{3}\right) = \frac{3\sqrt{3}}{2}$.

68. Since $|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$ we find that $f(x) = x|x| = \begin{cases} -x^2 & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases} \Rightarrow f'(x) = \begin{cases} -2x & \text{if } x < 0 \\ 2x & \text{if } x > 0 \end{cases} \Rightarrow$

$f''(x) = \begin{cases} -2 & \text{if } x < 0 \\ 2 & \text{if } x > 0 \end{cases}$

So f is concave downward on $(-\infty, 0)$ and concave upward on $(0, \infty)$. Therefore $(0, 0)$ is an inflection point of f. To show that f'' does not exist at 0, observe that if we define $f'(0) = 0$, then $f'(x) = 2|x|$, and the result of Example 6 in Section 2.1 tells us that $f''(0)$ does not exist.