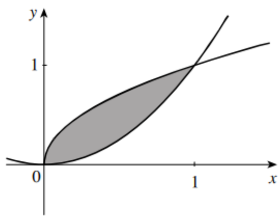
5.1 Areas Between Curves

17. Solve $\sqrt{x} = x^2 \Leftrightarrow x = x^4 \Leftrightarrow x\left(x^3 - 1\right) = 0$, giving (0, 0) and (1, 1) as the points of intersection. Thus,

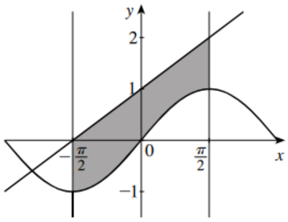
$A = \int_0^1 \left(\sqrt{x} - x^2\right) dx = \frac{2}{3}x^{3/2} - \frac{1}{3}x^3\Big|_0^1 = \frac{1}{3}$.



32. $A = \int_{-\pi/2}^{\pi/2} \left[\left(\frac{2}{\pi}x + 1\right) - \sin x\right] dx$

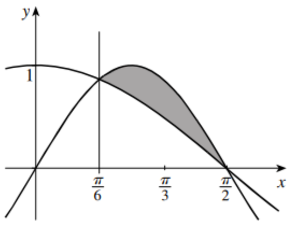
$= 2\int_0^{\pi/2} 1\, dx = \pi$

because $f(x) = \frac{2}{\pi}x - \sin x$ is odd.

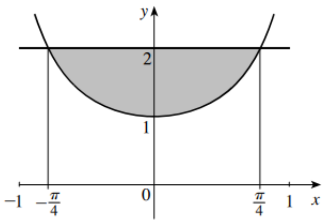


33. $A = \int_{\pi/6}^{\pi/2} (\sin 2x - \cos x)\, dx = \left(-\frac{1}{2}\cos 2x - \sin x\right)\Big|_{\pi/6}^{\pi/2}$

$= \left(\frac{1}{2} - 1\right) - \left(-\frac{1}{2}\cdot\frac{1}{2} - \frac{1}{2}\right) = \frac{1}{4}$



35. $A = 2\int_0^{\pi/4} \left(2 - \sec^2 x\right) dx = 2(2x - \tan x)|_0^{\pi/4} = \pi - 2$



5.2 Volumes: Disks, Washers, and Cross-Sections

11. $V = \pi\int_0^1 \left[\left(\frac{3}{2} - x^3\right)^2 - \left(\frac{3}{2} - x\right)^2\right] dx = \pi\int_0^1 \left(x^6 - 3x^3 - x^2 + 3x\right) dx = \pi\left(\frac{1}{7}x^7 - \frac{3}{4}x^4 - \frac{1}{3}x^3 + \frac{3}{2}x^2\right)\Big|_0^1 = \frac{47\pi}{84}$

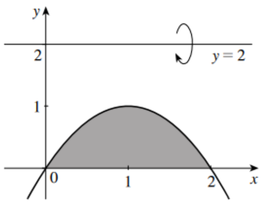
12. $V = \pi\int_0^1 \left[\left(\frac{3}{2} - y\right)^2 - \left(\frac{3}{2} - y^{1/3}\right)^2\right] dy = \pi\int_0^1 \left(y^2 - 3y - y^{2/3} + 3y^{1/3}\right) dy = \pi\left(\frac{1}{3}y^3 - \frac{3}{2}y^2 - \frac{3}{5}y^{5/3} + \frac{9}{4}y^{4/3}\right)\Big|_0^1$

$= \frac{29\pi}{60}$

33. $V = \pi \int_0^2 \left[(2-0)^2 - \left(2 + x^2 - 2x\right)^2 \right] dx$

$= \pi \int_0^2 \left(-x^4 + 4x^3 - 8x^2 + 8x\right) dx$

$= \pi \left(-\frac{1}{5}x^5 + x^4 - \frac{8}{3}x^3 + 4x^2\right)\Big|_0^2 = \frac{64\pi}{15}$



35. $V = \pi \int_{-2}^2 \left\{ (5-0)^2 - \left[\left(5 + x^2 - 4\right)\right]^2 \right\} dx$

$= 2\pi \int_0^2 \left[25 - \left(x^2 + 1\right)^2 \right] dx$

$= 2\pi \int_0^2 \left(-x^4 - 2x^2 + 24\right) dx$

$= 2\pi \left(-\frac{1}{5}x^5 - \frac{2}{3}x^3 + 24x\right)\Big|_0^2 = \frac{1088\pi}{15}$