7. 2

4. $\int_0^{\pi/2} \sqrt{\cos x}\sin^3 x\,dx = \int_0^{\pi/2} \cos^{1/2} x \cdot \sin^2 x \cdot \sin x\,dx = \int_0^{\pi/2} \cos^{1/2} x \left(1 - \cos^2 x\right) \sin x\,dx$

$= \int_0^{\pi/2} \left(\cos^{1/2} x - \cos^{5/2} x\right) \sin x\,dx$

Let $u = \cos x$, so $du = -\sin x\,dx$, $x = 0 \Rightarrow u = 1$, and $x = \frac{\pi}{2} \Rightarrow u = 0$. Then

$\int_0^{\pi/2} \sqrt{\cos x}\sin^3 x\,dx = -\int_1^0 \left(u^{1/2} - u^{5/2}\right) du = -\frac{2}{3}u^{3/2} + \frac{2}{7}u^{7/2}\Big|_1^0 = -\left(-\frac{2}{3} + \frac{2}{7}\right) = \frac{8}{21}.$

7. $$\int_0^{\pi} \cos^2 \frac{x}{2}\,dx = \int_0^{\pi} \frac{1+\cos x}{2}\,dx = \frac{x + \sin x}{2}\Big|_0^{\pi} = \frac{\pi}{2}$$

22. $\int \tan^5 x \sec^3 x\,dx = \int \tan^4 x \sec^2 x \sec x \tan x\,dx = \int \left(\sec^2 x - 1\right)^2 \sec^2 x \sec x \tan x\,dx$

$= \int \left(\sec^6 x - 2\sec^4 x + \sec^2 x\right) \sec x \tan x\,dx = \frac{1}{7}\sec^7 x - \frac{2}{5}\sec^5 x + \frac{1}{3}\sec^3 x + C$

24. $\int_0^{\pi/4} \sec^2 x \tan^2 x\,dx = \frac{1}{3}\tan^3 x\Big|_0^{\pi/4} = \frac{1}{3}$

39. $\int_0^{\pi/2} \sin x \cos 2x\,dx = \frac{1}{2}\int_0^{\pi/2} [\sin(1-2)x + \sin(1+2)x]\,dx = \frac{1}{2}\int_0^{\pi/2} (-\sin x + \sin 3x)\,dx$

$= \frac{1}{2}\left(\cos x - \frac{1}{3}\cos 3x\right)\Big|_0^{\pi/2} = \frac{1}{2}\left(0 - \frac{2}{3}\right) = -\frac{1}{3}$

42. $\displaystyle\int \frac{\sin^3 x}{\sec^2 x}\,dx = \int \sin^2 x \cos^2 x \sin x\,dx = \int \left(1 - \cos^2 x\right)\cos^2 x \sin x\,dx = \int \left(\cos^2 x - \cos^4 x\right) \sin x\,dx$

$= -\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x + C$

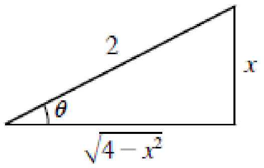
44. $\displaystyle\int_0^{\pi/2} \frac{\sin t}{1+\cos t}\,dt$. Let $u = 1 + \cos t$, so $du = -\sin t\,dt$, $t = 0 \Rightarrow u = 2$, and $t = \frac{\pi}{2} \Rightarrow u = 1$. Then

$\displaystyle\int_0^{\pi/2} \frac{\sin t}{1+\cos t}\,dt = -\int_2^1 \frac{du}{u} = -\ln u|_2^1 = -\ln 1 + \ln 2 = \ln 2.$

47. $\displaystyle\int \frac{1-\tan^2 x}{\sec^2 x}\,dx = \int \frac{1 - \left(\sec^2 x - 1\right)}{\sec^2 x}\,dx = \int \frac{2 - \sec^2 x}{\sec^2 x}\,dx = \int \left(2\cos^2 x - 1\right) dx = \int \cos 2x\,dx = \frac{1}{2}\sin 2x + C$

7. 3

3. $\int x\sqrt{4-x^2}\,dx$. Let $x = 2\sin\theta$, so $dx = 2\cos\theta\,d\theta$ and
$\sqrt{4-x^2} = \sqrt{4-4\sin^2\theta} = 2\cos\theta$. Then

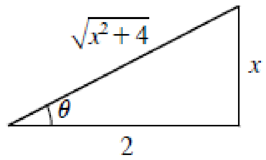


$$\int x\sqrt{4-x^2}\,dx = \int (2\sin\theta)(2\cos\theta)(2\cos\theta)\,d\theta = 8\int \cos^2\theta\sin\theta d\theta$$

$$= -\frac{8\cos^3\theta}{3} + C = -\frac{8}{3}\left(\frac{\sqrt{4-x^2}}{2}\right)^3 + C = -\frac{\left(4-x^2\right)^{3/2}}{3} + C$$

Note that the problem can be solved more easily using the substitution $u = 4 - x^2$.

7. $\int \frac{dx}{x^2\sqrt{x^2+4}}$. Let $x = 2\tan\theta$, so $dx = 2\sec^2\theta\,d\theta$ and
$\sqrt{x^2+4} = \sqrt{4\tan^2\theta+4} = 2\sqrt{\tan^2\theta+1} = 2\sec\theta$. Then



$$\int \frac{dx}{x^2\sqrt{x^2+4}} = \int \frac{2\sec^2\theta\,d\theta}{(4\tan^2\theta)(2\sec\theta)} = \frac{1}{4}\int \frac{\sec\theta}{\tan^2\theta}\,d\theta = \frac{1}{4}\int (\sin\theta)^{-2}\cos\theta\,d\theta$$

$$= -\frac{1}{4\sin\theta} + C = -\frac{\sqrt{x^2+4}}{4x} + C$$

22. $I = \int_2^4 \frac{\sqrt{x^2-4}}{x^4}\,dx$. Let $x = 2\sec\theta$, so $dx = 2\sec\theta\tan\theta\,d\theta$,
$\sqrt{x^2-4} = \sqrt{4\sec^2\theta-4} = 2\tan\theta$, $x = 2 \Rightarrow \theta = 0$, and $x = 4 \Rightarrow \theta = \frac{\pi}{3}$. Then

$$I = \int_2^4 \frac{\sqrt{x^2-4}}{x^4}\,dx = \int_0^{\pi/3} \frac{2\tan\theta\,(2\sec\theta\tan\theta\,d\theta)}{(2\sec\theta)^4} = \frac{1}{4}\int_0^{\pi/3}\sin^2\theta\cos\theta\,d\theta = \frac{1}{12}\sin^3\theta\Big|_0^{\pi/3} = \frac{1}{12}\left(\frac{\sqrt{3}}{2}\right)^3 = \frac{\sqrt{3}}{32}.$$

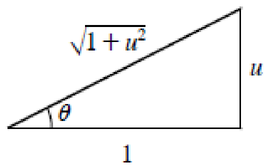
25. $I = \int e^x\sqrt{4-e^{2x}}\,dx$. Let $u = e^x$, so $du = e^x\,dx$. Then $I = \int \sqrt{4-u^2}\,du$. Next, let $u = 2\sin\theta$, so $du = 2\cos\theta\,d\theta$ and
$\sqrt{4-u^2} = \sqrt{4-4\sin^2\theta} = 2\sqrt{1-\sin^2\theta} = 2\cos\theta$. Then

$$I = \int (2\cos\theta)(2\cos\theta\,d\theta) = 4\int\cos^2\theta\,d\theta = 4\int\frac{1+\cos 2\theta}{2}\,d\theta = 2\theta + \sin 2\theta + C = 2\theta + 2\sin\theta\cos\theta + C$$

$$= 2\left[\sin^{-1}\left(\tfrac{1}{2}u\right) + \tfrac{1}{2}u\left(\tfrac{1}{2}\sqrt{4-u^2}\right)\right] + C$$

Therefore, $\int e^x\sqrt{4-e^{2x}}\,dx = 2\sin^{-1}\left(\frac{1}{2}e^x\right) + \frac{1}{2}e^x\sqrt{4-e^{2x}} + C$.

26. $I = \int e^t \sqrt{1 + e^{2t}}\, dt$. Let $u = e^t$, so $du = e^t\, dt$. Then $I = \int \sqrt{1 + u^2}\, du$. Next, let $u = \tan\theta$, so $du = \sec^2\theta\, d\theta$ and $\sqrt{1+u^2} = \sqrt{1 + \tan^2\theta} = \sec\theta$. Then



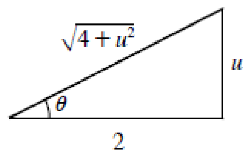
$$I = \int \sec\theta \sec^2\theta\, d\theta = \int \sec^3\theta\, d\theta$$

$= \frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta + \tan\theta| + C$ (see Example 5 in Section 7.1)

$$= \tfrac{1}{2}\sqrt{1+u^2}u + \tfrac{1}{2}\ln\left|\sqrt{1+u^2} + u\right| + C$$

Thus, $\int e^t \sqrt{1+e^{2t}}\, dt = \frac{1}{2}\left[e^t\sqrt{1+e^{2t}} + \ln\left(\sqrt{1+e^{2t}} + e^t\right)\right] + C$.

31. $I = \displaystyle\int \frac{dx}{(x^2+4x+8)^2}$. Observe that



$x^2 + 4x + 8 = \left[x^2 + 4x + 2^2\right] + 8 - 4 = 4 + (x+2)^2$, so $I = \displaystyle\int \frac{dx}{\left[4 + (x+2)^2\right]^2}$. Let

$u = x + 2$, so $du = dx$. Then $I = \displaystyle\int \frac{du}{(4+u^2)^2}$. Next, let $u = 2\tan\theta$, so $du = 2\sec^2\theta\, d\theta$

and $4 + u^2 = 4 + 4\tan^2\theta = 4\sec^2\theta$. Then

$$I = \int \frac{2\sec^2\theta\, d\theta}{(4\sec^2\theta)^2} = \frac{1}{8}\int \frac{d\theta}{\sec^2\theta} = \frac{1}{8}\int \cos^2\theta\, d\theta = \frac{1}{8}\int \frac{1+\cos 2\theta}{2}\, d\theta = \tfrac{1}{16}\left(\theta + \tfrac{1}{2}\sin 2\theta\right) + C$$

$$= \tfrac{1}{16}\theta + \tfrac{1}{16}\sin\theta\cos\theta + C = \frac{1}{16}\tan^{-1}\left(\frac{u}{2}\right) + \frac{1}{16}\frac{u}{\sqrt{4+u^2}}\frac{2}{\sqrt{4+u^2}} + C = \frac{1}{16}\tan^{-1}\left(\frac{u}{2}\right) + \frac{u}{8\left(4+u^2\right)} + C$$

Therefore, $I = \dfrac{1}{16}\tan^{-1}\left(\dfrac{x+2}{2}\right) + \dfrac{x+2}{8\left(x^2+4x+8\right)} + C$.