

First Midterm

考試注意事項：

1. 答案紙直行對折，兩直欄書寫作答。
2. 無清楚計算過程，不予計分。
3. 此次考試，禁用羅必達規則 (No use of L'Hôpital's rule in this test!)。

試題：

1. (15%) Let

$$f(x) = \begin{cases} x \cot kx & \text{if } x < 0 \\ x^2 + c & \text{if } x \geq 0. \end{cases}$$

Find the value of c that will make f continuous at $x = 0$.

2. (15%) Use the definition of the derivative of a function,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h},$$

to find $f'(x)$ for $f(x) = \sqrt{x}$.

3. (15%) Find the derivative of $y = \frac{\sin x}{1 - \cos x}$.
4. (15%) Find $\frac{dy}{dx}$ at the point $(\frac{\pi}{2}, \pi)$ if $x \sin y - y \cos 2x = 2x$.
5. (10%) Find the limit if it exists:

$$\lim_{x \rightarrow 0} \frac{\tan^2 x}{x}$$

6. (10%) Let

$$f(x) = \begin{cases} \frac{x^2 - x - 2}{x - 2} & \text{if } x \neq 2 \\ 1 & \text{if } x = 2. \end{cases}$$

Show that f has a removable discontinuity at $x = 2$. Redefine f at 2 so that it is continuous everywhere.

7. (10%) Find an equation of the tangent line to the graph of

$$f(x) = \frac{(2x^2 + 1)(x^3 - 1)}{x^2 + 4}.$$

at the point where $x = 1$.

8. (10%) Find an equation of the tangent line at the point on the graph of $y = x^2 \sin 3x$, where $x = \pi/2$.