12. 1

39. $\lim_{t\to 2}\left[\sqrt{t}\,\mathbf{i}+\frac{t^2-4}{t-2}\mathbf{j}+\frac{t}{t^2+1}\mathbf{k}\right]=\lim_{t\to 2}\sqrt{t}\,\mathbf{i}+\lim_{t\to 2}(t+2)\,\mathbf{j}+\lim_{t\to 2}\frac{t}{t^2+1}\mathbf{k}=\sqrt{2}\,\mathbf{i}+4\mathbf{j}+\frac{2}{5}\mathbf{k}$

42. $\lim_{t\to -\infty}\left[\frac{t-1}{2t+1}\mathbf{i}+e^{2t}\mathbf{j}+\tan^{-1}t\,\mathbf{k}\right]=\lim_{t\to -\infty}\frac{1-\frac{1}{t}}{2+\frac{1}{t}}\mathbf{i}+\lim_{t\to -\infty}e^{2t}\mathbf{j}+\lim_{t\to -\infty}\tan^{-1}t\,\mathbf{k}=\frac{1}{2}\mathbf{i}-\frac{\pi}{2}\mathbf{k}$

47. Since $f(t)=e^{-t}$ is continuous on $(-\infty,\infty)$, $g(t)=\cos\sqrt{4-t}$ is continuous on $(-\infty,4]$, and $h(t)=1/\left(t^2-1\right)$ is continuous on $(-\infty,-1)$, $(-1,1)$, and $(1,\infty)$, we see that $\mathbf{r}$ is continuous on $(-\infty,-1)$, $(-1,1)$, and $(1,4]$.

12. 2

4. $\mathbf{r}(t) = \langle t\cos t, t\sin t, \tan t\rangle \Rightarrow \mathbf{r}'(t) = \left\langle \cos t - t\sin t, \sin t + t\cos t, \sec^2 t\right\rangle \Rightarrow$
$\mathbf{r}''(t) = \left\langle -t\cos t - 2\sin t, 2\cos t - t\sin t, 2\sec^2 t\tan t\right\rangle$

19. $\mathbf{r}(t) = 2\sin 2t\,\mathbf{i} + 3\cos 2t\,\mathbf{j} + 3\mathbf{k} \Rightarrow \mathbf{r}'(t) = 4\cos 2t\,\mathbf{i} - 6\sin 2t\,\mathbf{j} \Rightarrow \mathbf{r}'\left(\frac{\pi}{6}\right) = 2\mathbf{i} - 3\sqrt{3}\mathbf{j}$, so
$\left|\mathbf{r}'\left(\frac{\pi}{6}\right)\right| = \sqrt{2^2 + \left(-3\sqrt{3}\right)^2} = \sqrt{31}$, and therefore $\mathbf{T}\left(\frac{\pi}{6}\right) = \dfrac{\mathbf{r}'\left(\frac{\pi}{6}\right)}{\left|\mathbf{r}'\left(\frac{\pi}{6}\right)\right|} = \frac{1}{\sqrt{31}}\left(2\mathbf{i} - 3\sqrt{3}\mathbf{j}\right) = \frac{2\sqrt{31}}{31}\mathbf{i} - \frac{3\sqrt{93}}{31}\mathbf{j}$.

30. $\int_1^2 \left[\sqrt{t-1}\,\mathbf{i} + \frac{1}{\sqrt{t}}\mathbf{j} + (2t-1)^5\,\mathbf{k}\right]dt = \left[\frac{2}{3}(t-1)^{3/2}\,\mathbf{i} + 2t^{1/2}\mathbf{j} + \frac{1}{12}(2t-1)^6\,\mathbf{k}\right]_1^2 = \left(\frac{2}{3}\mathbf{i} + 2\sqrt{2}\mathbf{j} + \frac{243}{4}\mathbf{k}\right) -$
$\left(2\mathbf{j} + \frac{1}{12}\mathbf{k}\right) = \frac{2}{3}\mathbf{i} + 2\left(\sqrt{2} - 1\right)\mathbf{j} + \frac{182}{3}\mathbf{k}$

13.2

4. Along $y = 0$, $\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4} = \lim_{x\to 0} \frac{0}{x^2} = \lim_{x\to 0} 0 = 0$. Along $x = y^2$, $\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4} = \lim_{y\to 0} \frac{y^4}{2y^4} = \lim_{y\to 0} \frac{1}{2} = \frac{1}{2}$. Because these two limits are not equal, the given limit does not exist.

11. Along the z-axis, $\lim_{(x,y,z)\to(0,0,0)} \frac{xz^2+2y^2}{x^2+2y^2+z^4} = \lim_{z\to 0} \frac{0}{z^4} = \lim_{z\to 0} 0 = 0$. Let $C: x = t^2, y = t^2, z = t$. Then along C, $\lim_{(x,y,z)\to(0,0,0)} \frac{xz^2+2y^2}{x^2+2y^2+z^4} = \lim_{t\to 0} \frac{t^4+2t^4}{t^4+2t^4+t^4} = \lim_{t\to 0} \frac{3}{4} = \frac{3}{4}$. Because these two limits are not equal, the given limit does not exist.

15. $\lim_{(x,y)\to(1,2)} \frac{2x^2-3y^3+4}{3-xy} = \frac{2(1)^2-3(2)^3+4}{3-(1)(2)} = -18$

32. The function f is a rational function whose denominator $x^2+y^2 = 0 \Rightarrow (x,y) = (0,0)$. Thus, f is continuous on $\{(x,y) \mid (x,y) \neq (0,0)\}$.