

## Final Test

## 考試注意事項：

1. 答案紙直行對折，兩直欄書寫作答。題號請標示清楚在答案紙上。
2. 無清楚計算過程，不予計分。
3. 沒依題目要求，不予計分。

## 試題：

1. (15%) Show that  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - y^2}{x^2 + y^2}$  does not exist.
2. (15%) Find the relative extrema of  $f(x, y) = x^3 + y^2 - 2xy + 7x - 8y + 2$ .
3. (15%) Evaluate the integral by reversing the order of integration:

$$\int_0^4 \int_{\sqrt{x}}^2 \sin y^3 \, dy dx.$$

4. (15%) Use the method of **change of variables** to evaluate

$$\iint_R (x + y) \, dA,$$

where  $R$  is the parallelogram with vertices  $(0, 0)$ ,  $(2, 0)$ ,  $(3, 1)$ , and  $(1, 1)$ .

5. (10%) Let  $w = x^2y + y^2z^3$ , where  $x = r \cos s$ ,  $y = r \sin s$  and  $z = re^s$ . Use the method of the **chain rule** to find the value of  $\partial w / \partial s$  when  $r = 1$  and  $s = 0$ .
6. (10%) Find the directional derivative of  $f(x, y) = e^x \cos 2y$  at the point  $(0, \frac{\pi}{4})$  in the direction  $\mathbf{v} = 2\mathbf{i} + 3\mathbf{j}$ .
7. (10%) Find equations of the tangent plane and normal line to the graph of the function  $f$  defined by  $f(x, y) = 4x^2 + y^2 + 2$  at the point where  $x = 1$  and  $y = 1$ .
8. (10%) Use **Lagrange multipliers** to find the maximum and minimum values of the function  $f(x, y) = x^2 - 2y$  subject to  $x^2 + y^2 = 9$ .