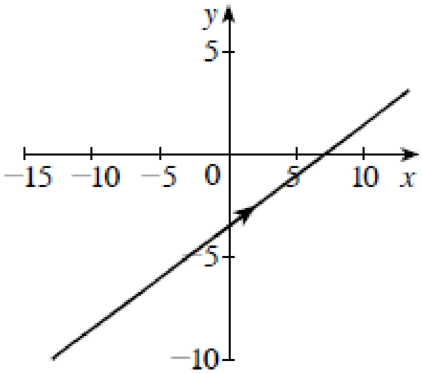
10.2

1. a. $\left.\begin{array}{r} x = 2t + 1 \\ y + 3 = t \end{array}\right\} \Rightarrow x = 2(y+3) + 1$ or

$x - 2y - 7 = 0$

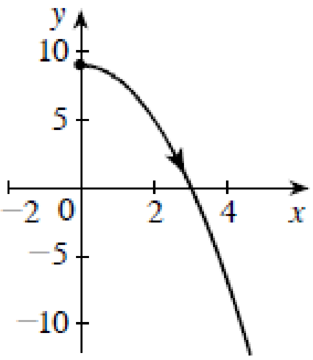
b.



The orientation is found by observing that as t increases, so does x.

3. a. $\left.\begin{array}{r} x = \sqrt{t} \\ y = 9 - t \end{array}\right\} \Rightarrow y = 9 - x^2, x \geq 0$

b.



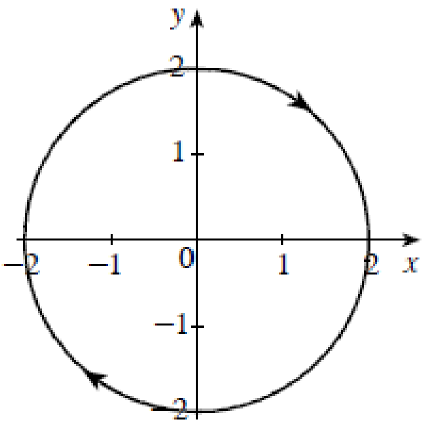
The orientation is found by observing that as t increases, so does x.

9. a. $\left.\begin{array}{l} x = 2\sin\theta \\ y = 2\cos\theta \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \sin\theta = \frac{1}{2}x \\ \cos\theta = \frac{1}{2}y \end{array}\right\} \Rightarrow$

$\left(\frac{1}{2}x\right)^2 + \left(\frac{1}{2}y\right)^2 = \sin^2\theta + \cos^2\theta = 1$, so

$x^2 + y^2 = 4$.

b.



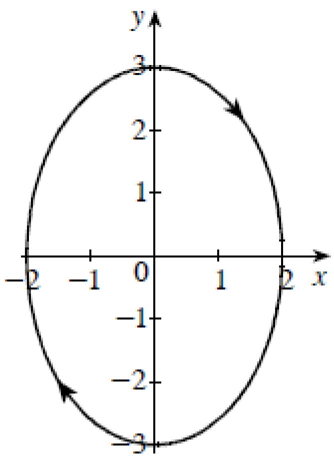
Observe that as θ increases from 0 to 2π, the curve C is traced once in a clockwise direction starting from the point $(0, 2)$.

11. a. $\left.\begin{array}{l} x = 2\sin\theta \\ y = 3\cos\theta \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \sin\theta = \frac{1}{2}x \\ \cos\theta = \frac{1}{3}y \end{array}\right\} \Rightarrow$

$\left(\frac{1}{2}x\right)^2 + \left(\frac{1}{3}y\right)^2 = \sin^2\theta + \cos^2\theta = 1 \Rightarrow$

$\frac{1}{4}x^2 + \frac{1}{9}y^2 = 1$

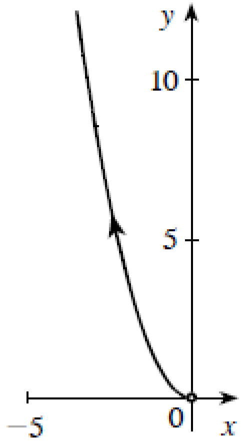
b.



21. a. $\left.\begin{array}{l} x = -e^t \\ y = e^{2t} \end{array}\right\} \Rightarrow y = \left(e^t\right)^2 = (-x)^2 \Rightarrow y = x^2,$

$x < 0$

b.



10.3

4. $x = e^{2t}$, $y = \ln t \Rightarrow \frac{dx}{dt} = 2e^{2t}$ and $\frac{dy}{dt} = 1/t$. The slope of the tangent line at $t = 1$ is

$$\left.\frac{dy}{dx}\right|_{t=1} = \left.\frac{dy/dt}{dx/dt}\right|_{t=1} = \left.\frac{1/t}{2e^{2t}}\right|_{t=1} = \frac{1}{2e^2}.$$

6. $x = 2(\theta - \sin\theta)$, $y = 2(1-\cos\theta) \Rightarrow \frac{dx}{d\theta} = 2(1-\cos\theta)$ and $\frac{dy}{d\theta} = 2\sin\theta$. The slope of the tangent line at $\theta = \frac{\pi}{6}$ is

$$\left.\frac{dy}{dx}\right|_{\theta=\pi/6} = \left.\frac{dy/d\theta}{dx/d\theta}\right|_{\theta=\pi/6} = \left.\frac{2\sin\theta}{2(1-\cos\theta)}\right|_{\theta=\pi/6} = \frac{1}{2\left(1-\frac{\sqrt{3}}{2}\right)} = \frac{1}{2-\sqrt{3}} = 2+\sqrt{3}.$$

8. $x = \theta\cos\theta$, $y = \theta\sin\theta \Rightarrow \frac{dx}{d\theta} = \cos\theta - \theta\sin\theta$ and $\frac{dy}{d\theta} = \sin\theta + \theta\cos\theta$. The point of tangency is $\left(x\left(\frac{\pi}{2}\right), y\left(\frac{\pi}{2}\right)\right) = \left(0, \frac{\pi}{2}\right)$ and the slope of the tangent line is

$$\left.\frac{dy}{dx}\right|_{\theta=\pi/2} = \left.\frac{dy/d\theta}{dx/d\theta}\right|_{\theta=\pi/2} = \left.\frac{\sin\theta+\theta\cos\theta}{\cos\theta-\theta\sin\theta}\right|_{\theta=\pi/2} = -\frac{2}{\pi},$$ so an equation is $y - \frac{\pi}{2} = -\frac{2}{\pi}(x-0)$ or $y = -\frac{2}{\pi}x + \frac{\pi}{2}$.

20. $x = \sin 2t$, $y = \cos 2t \Rightarrow \frac{dx}{dt} = 2\cos 2t$ and $\frac{dy}{dt} = -2\sin 2t$, so $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-2\sin 2t}{2\cos 2t} = -\tan 2t$ and

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{d}{dt}(-\tan 2t)}{2\cos 2t} = \frac{-2\sec^2 2t}{2\cos 2t} = -\sec^3 2t.$$

22. $x = e^{-t}$, $y = e^{2t} \Rightarrow \frac{dx}{dt} = -e^{-t}$ and $\frac{dy}{dt} = 2e^{2t}$, so $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2e^{2t}}{-e^{-t}} = -2e^{3t}$ and

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{d}{dt}\left(-2e^{3t}\right)}{-e^{-t}} = \frac{-6e^{3t}}{-e^{-t}} = 6e^{4t}.$$

24. $x = \sqrt{t^2+1}$, $y = t\ln t \Rightarrow \frac{dx}{dt} = \frac{t}{\sqrt{t^2+1}}$ and $\frac{dy}{dt} = \ln t + 1$, so $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\ln t + 1}{t\big/\sqrt{t^2+1}} = \frac{\sqrt{t^2+1}(\ln t+1)}{t}$. Next,

$$\frac{d}{dt}\left[\frac{\left(t^2+1\right)^{1/2}(\ln t+1)}{t}\right] = \frac{t\left[\frac{1}{2}\left(t^2+1\right)^{-1/2}(2t)(\ln t+1) + \frac{\left(t^2+1\right)^{1/2}}{t}\right] - \left(t^2+1\right)^{1/2}(\ln t+1)}{t^2} = \frac{t^2-\ln t}{t^2\sqrt{t^2+1}},$$

so $$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{t^2-\ln t}{t^2\sqrt{t^2+1}} \cdot \frac{\sqrt{t^2+1}}{t} = \frac{t^2-\ln t}{t^3}.$$

28. $x = \int_1^t \frac{\sin u}{u}\,du$, $y = \int_2^{\ln t} e^u\,du$. Using the Fundamental Theorem of Calculus, Part I, we have

$\frac{dx}{dt} = \frac{d}{dt}\int_1^t \frac{\sin u}{u}\,du = \frac{\sin t}{t}$ and $\frac{dy}{dt} = \frac{d}{dt}\int_2^{\ln t} e^u\,du = e^{\ln t}\cdot\frac{d}{dt}\ln t = \frac{t}{t} = 1$, so $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{1}{\frac{\sin t}{t}} = \frac{t}{\sin t}$ and

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{1}{\frac{\sin t}{t}}\cdot\frac{d}{dt}\left(\frac{t}{\sin t}\right) = \frac{t}{\sin t}\cdot\frac{\sin t - t\cos t}{(\sin t)^2} = \frac{t(\sin t - t\cos t)}{\sin^3 t}.$$

11.1

27. $2\mathbf{a} = 2\langle -1, 2\rangle = \langle -2, 4\rangle$, $\mathbf{a}+\mathbf{b} = \langle -1, 2\rangle + \langle 3, 1\rangle = \langle 2, 3\rangle$, $\mathbf{a}-\mathbf{b} = \langle -1, 2\rangle - \langle 3, 1\rangle = \langle -4, 1\rangle$, $2\mathbf{a}+\mathbf{b} = 2\langle -1, 2\rangle + \langle 3, 1\rangle = \langle 1, 5\rangle$, and $|2\mathbf{a}+\mathbf{b}| = |\langle 1, 5\rangle| = \sqrt{1^2+5^2} = \sqrt{26}$.

29. $2\mathbf{a} = 2(3\mathbf{i}-2\mathbf{j}) = 6\mathbf{i}-4\mathbf{j}$, $\mathbf{a}+\mathbf{b} = (3\mathbf{i}-2\mathbf{j})+2\mathbf{i} = 5\mathbf{i}-2\mathbf{j}$, $\mathbf{a}-\mathbf{b} = (3\mathbf{i}-2\mathbf{j})-2\mathbf{i} = \mathbf{i}-2\mathbf{j}$, $2\mathbf{a}+\mathbf{b} = 2(3\mathbf{i}-2\mathbf{j})+2\mathbf{i} = 8\mathbf{i}-4\mathbf{j}$, and $|2\mathbf{a}+\mathbf{b}| = |8\mathbf{i}-4\mathbf{j}| = \sqrt{8^2+4^2} = 4\sqrt{5}$.

31. $2\mathbf{a} = 2\left(\frac{1}{2}\mathbf{i}+\frac{3}{2}\mathbf{j}\right) = \mathbf{i}+3\mathbf{j}$, $\mathbf{a}+\mathbf{b} = \left(\frac{1}{2}\mathbf{i}+\frac{3}{2}\mathbf{j}\right)+\left(\frac{3}{4}\mathbf{i}-\frac{1}{4}\mathbf{j}\right) = \frac{5}{4}\mathbf{i}+\frac{5}{4}\mathbf{j}$, $\mathbf{a}-\mathbf{b} = \left(\frac{1}{2}\mathbf{i}+\frac{3}{2}\mathbf{j}\right)-\left(\frac{3}{4}\mathbf{i}-\frac{1}{4}\mathbf{j}\right) = -\frac{1}{4}\mathbf{i}+\frac{7}{4}\mathbf{j}$, $2\mathbf{a}+\mathbf{b} = 2\left(\frac{1}{2}\mathbf{i}+\frac{3}{2}\mathbf{j}\right)+\left(\frac{3}{4}\mathbf{i}-\frac{1}{4}\mathbf{j}\right) = \frac{7}{4}\mathbf{i}+\frac{11}{4}\mathbf{j}$, and $|2\mathbf{a}+\mathbf{b}| = \left|\frac{7}{4}\mathbf{i}+\frac{11}{4}\mathbf{j}\right| = \sqrt{\left(\frac{7}{4}\right)^2+\left(\frac{11}{4}\right)^2} = \frac{\sqrt{170}}{4}$.

33. $2\mathbf{a}-3\mathbf{b} = 2(2\mathbf{i})-3(-6\mathbf{j}) = 4\mathbf{i}+18\mathbf{j}$, $\frac{1}{2}\mathbf{a}+\frac{1}{3}\mathbf{b} = \frac{1}{2}(2\mathbf{i})+\frac{1}{3}(-6\mathbf{j}) = \mathbf{i}-2\mathbf{j}$

35. $a\mathbf{u}+b\mathbf{v} = \mathbf{w} \Rightarrow a\langle -1, 3\rangle + b\langle 2, 4\rangle = \langle 6, 4\rangle \Leftrightarrow \langle -a+2b, 3a+4b\rangle = \langle 6, 4\rangle \Leftrightarrow \left.\begin{array}{r} -a+2b=6 \\ 3a+4b=4 \end{array}\right\} \Leftrightarrow a = -1.6$ and $b = 2.2$.

49. A unit vector in the direction of $\mathbf{b}$ is $\mathbf{u} = \dfrac{\mathbf{b}}{|\mathbf{b}|} = \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right\rangle$, and so $\mathbf{a} = 5\mathbf{u} = \left\langle \frac{5\sqrt{2}}{2}, \frac{5\sqrt{2}}{2}\right\rangle$.

53. $2\mathbf{a}-3\mathbf{b} = 2\langle -3, 4\rangle - 3\langle 1, 2\rangle = \langle -9, 2\rangle$ and a unit vector in the direction of this vector is $\mathbf{u} = \dfrac{2\mathbf{a}-3\mathbf{b}}{|2\mathbf{a}-3\mathbf{b}|} = \dfrac{\langle -9, 2\rangle}{\sqrt{(-9)^2+2^2}} = \left\langle -\frac{9\sqrt{85}}{85}, \frac{2\sqrt{85}}{85}\right\rangle$. Thus, the required vector is $3\mathbf{u} = \left\langle -\frac{27\sqrt{85}}{85}, \frac{6\sqrt{85}}{85}\right\rangle$.

11.2

33. Denote the point on the sphere by $A(1,3,5)$. Then its radius is $d(A,C)$, where $C(-1,2,4)$ is its center. Thus, $r=\sqrt{(-2)^2+(-1)^2+(-1)^2}=\sqrt{6}$. Therefore, the required equation is $(x+1)^2+(y-2)^2+(z-4)^2=6$.

37. $x^2+y^2+z^2-4x+6y=0$. Completing the squares in x and y, we obtain $\left[x^2-4x+(-2)^2\right]+\left[y^2+6y+(3)^2\right]+z^2=4+9$ $\Leftrightarrow (x-2)^2+(y+3)^2+z^2=13$. Thus, the sphere has center $(2,-3,0)$ and radius $\sqrt{13}$.

53. $\mathbf{a}=\langle -1,2,0\rangle$ and $\mathbf{b}=\langle 2,3,-1\rangle$, so $\mathbf{a}+\mathbf{b}=\langle 1,5,-1\rangle$,

$2\mathbf{a}-3\mathbf{b}=2\langle -1,2,0\rangle-3\langle 2,3,-1\rangle=\langle -2,4,0\rangle-\langle 6,9,-3\rangle=\langle -8,-5,3\rangle$,

$|3\mathbf{a}|=|3\langle -1,2,0\rangle|=|\langle -3,6,0\rangle|=\sqrt{(-3)^2+6^2+0^2}=3\sqrt{5}$,

$|-2\mathbf{b}|=|-2\langle 2,3,-1\rangle|=|\langle -4,-6,2\rangle|=\sqrt{(-4)^2+(-6)^2+2^2}=2\sqrt{14}$, and

$|\mathbf{a}-\mathbf{b}|=|\langle -1,2,0\rangle-\langle 2,3,-1\rangle|=|\langle -3,-1,1\rangle|=\sqrt{(-3)^2+(-1)^2+1^2}=\sqrt{11}$.

56. $\mathbf{a}=-2\mathbf{i}+4\mathbf{k}$ and $\mathbf{b}=2\mathbf{j}-\mathbf{k}$, so $\mathbf{a}+\mathbf{b}=-2\mathbf{i}+2\mathbf{j}+3\mathbf{k}$,

$2\mathbf{a}-3\mathbf{b}=2(-2\mathbf{i}+4\mathbf{k})-3(2\mathbf{j}-\mathbf{k})=(-4\mathbf{i}+8\mathbf{k})-(6\mathbf{j}-3\mathbf{k})=-4\mathbf{i}-6\mathbf{j}+11\mathbf{k}$,

$|3\mathbf{a}|=|3(-2\mathbf{i}+4\mathbf{k})|=|-6\mathbf{i}+12\mathbf{k}|=\sqrt{(-6)^2+12^2}=6\sqrt{5}$,

$|-2\mathbf{b}|=|-2(2\mathbf{j}-\mathbf{k})|=|-4\mathbf{j}+2\mathbf{k}|=\sqrt{(-4)^2+2^2}=2\sqrt{5}$, and

$|\mathbf{a}-\mathbf{b}|=|(-2\mathbf{i}+4\mathbf{k})-(2\mathbf{j}-\mathbf{k})|=|-2\mathbf{i}-2\mathbf{j}+5\mathbf{k}|=\sqrt{(-2)^2+(-2)^2+5^2}=\sqrt{33}$.

63. $\mathbf{a}=\langle 1,2,2\rangle$, so $|\mathbf{a}|=\sqrt{1^2+2^2+2^2}=3$.

a. $\mathbf{u}_1=\frac{1}{3}\mathbf{a}=\left\langle \frac{1}{3},\frac{2}{3},\frac{2}{3}\right\rangle$ **b.** $\mathbf{u}_2=\left\langle -\frac{1}{3},-\frac{2}{3},-\frac{2}{3}\right\rangle$

65. $\mathbf{a}=-\mathbf{i}+3\mathbf{j}-\mathbf{k}$, so $|\mathbf{a}|=\sqrt{(-1)^2+3^2+(-1)^2}=\sqrt{11}$.

a. $\mathbf{u}_1=\frac{1}{\sqrt{11}}(-\mathbf{i}+3\mathbf{j}-\mathbf{k})=-\frac{\sqrt{11}}{11}\mathbf{i}+\frac{3\sqrt{11}}{11}\mathbf{j}-\frac{\sqrt{11}}{11}\mathbf{k}$

b. $\mathbf{u}_2=\frac{\sqrt{11}}{11}\mathbf{i}-\frac{3\sqrt{11}}{11}\mathbf{j}+\frac{\sqrt{11}}{11}\mathbf{k}$

71. $\mathbf{a}=\langle 3,-1,2\rangle$ and $\mathbf{b}=\langle 1,0,-1\rangle$, so $\mathbf{a}-2\mathbf{b}=\langle 3,-1,2\rangle-2\langle 1,0,-1\rangle=\langle 1,-1,4\rangle$ and $|\mathbf{a}-2\mathbf{b}|=\sqrt{1^2+(-1)^2+4^2}=\sqrt{18}=3\sqrt{2}$. Therefore, a unit vector in the direction of $\mathbf{a}-2\mathbf{b}$ is $\mathbf{u}=\frac{1}{3\sqrt{2}}\langle 1,-1,4\rangle$, and the required vector is $2\mathbf{u}=\frac{2}{3\sqrt{2}}\langle 1,-1,4\rangle=\left\langle \frac{\sqrt{2}}{3},-\frac{\sqrt{2}}{3},\frac{4\sqrt{2}}{3}\right\rangle$.