

## 6.3 Exponential Functions

18.  $\lim_{x \rightarrow 0^+} \frac{1}{1 + e^{1/x}} = 0$  since  $e^{1/x} \rightarrow \infty$  as  $x \rightarrow 0^+$ .

24.  $f'(x) = \frac{d}{dx} (x^2 e^{-2x}) = x^2 \frac{d}{dx} (e^{-2x}) + e^{-2x} \frac{d}{dx} (x^2) = -2x^2 e^{-2x} + 2x e^{-2x} = 2x e^{-2x} (1 - x)$

26.  $h'(t) = \frac{d}{dt} \left( \frac{e^t - e^{-t}}{e^t + e^{-t}} \right) = \frac{d}{dt} \left( \frac{e^{2t} - 1}{e^{2t} + 1} \right) = \frac{(e^{2t} + 1)(2e^{2t}) - (e^{2t} - 1)(2e^{2t})}{(e^{2t} + 1)^2} = \frac{4e^{2t}}{(e^{2t} + 1)^2}$

28.  $g'(x) = \frac{d}{dx} (e^{-2x} \cos 3x) = e^{-2x} (-3 \sin 3x) + (\cos 3x) (-2e^{-2x}) = -e^{-2x} (3 \sin 3x + 2 \cos 3x)$

30.  $y' = \frac{d}{dx} (e^{-1/x}) = e^{-1/x} \left[ - \left( -\frac{1}{x^2} \right) \right] = \frac{e^{-1/x}}{x^2}$

32.  $h'(x) = \frac{d}{dx} (e^{2x} - e^{-3x})^5 = 5(e^{2x} - e^{-3x})^4 (2e^{2x} + 3e^{-3x})$

34.  $g'(x) = \frac{d}{dx} \ln(e^x + e^{-x}) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

36.  $h'(x) = \frac{d}{dx} [\tan(e^{2x} + \ln x)] = [\sec^2(e^{2x} + \ln x)] \left( 2e^{2x} + \frac{1}{x} \right) = \frac{1}{x} (2xe^{2x} + 1) \sec^2(e^{2x} + \ln x)$

38.  $y' = \frac{d}{dx} (e^{-x} \tan e^x) = -e^{-x} \tan e^x + (e^{-x} \sec^2 e^x) e^x = \frac{e^x \sec^2 e^x - \tan e^x}{e^x}$

42.  $e^{xy} - x^2 + y^2 = 5 \Rightarrow e^{xy} (y + xy') - 2x + 2yy' = 0 \Leftrightarrow y' (xe^{xy} + 2y) + ye^{xy} - 2x = 0 \Leftrightarrow y' = \frac{2x - ye^{xy}}{xe^{xy} + 2y}$

44.  $x \ln y + e^{-x} - ye^y = 0 \Rightarrow \ln y + \frac{x}{y} y' - e^{-x} - (e^y + ye^y) y' = 0 \Leftrightarrow y' \left( \frac{x}{y} - e^y - ye^y \right) - e^{-x} + \ln y = 0 \Leftrightarrow$   
 $y' = \frac{y(e^{-x} - \ln y)}{x - ye^y - y^2 e^y}$

49.  $y = xe^{-x} \Rightarrow y' = e^{-x} - xe^{-x} = (1 - x)e^{-x} \Rightarrow y'|_1 = 0$ . Thus, the slope of the required tangent line is  $m = 0$ , and an equation of the line is  $y - e^{-1} = 0(x - 1)$  or  $y = 1/e$ .

50.  $xe^y + 2x + y = 3 \Rightarrow e^y + xe^y y' + 2 + y' = 0$ . Substituting  $x = 1$  and  $y = 0$  into this equation gives  $1 + y' + 2 + y' = 0$  or  $y'|_{(1,0)} = -\frac{3}{2}$ , so the slope of the required tangent line is  $m = -\frac{3}{2}$ , and an equation is  $y - 0 = -\frac{3}{2}(x - 1)$  or  $y = -\frac{3}{2}x + \frac{3}{2}$ .

85. Let  $u = -x^2$ , so  $\int xe^{-x^2} dx = -\frac{1}{2}e^{-x^2} + C$ .

90.  $I = \int \frac{e^{-x}}{1 + e^{-x}} dx$ . Let  $u = 1 + e^{-x}$ , so  $du = -e^{-x} dx$ . Then

$$\int \frac{e^{-x}}{1 + e^{-x}} dx = -\int \frac{du}{u} = -\ln|u| + C = -\ln(1 + e^{-x}) + C.$$

91.  $I = \int \frac{e^x + e^{-x}}{e^x - e^{-x}} dx$ . Let  $u = e^x - e^{-x}$ , so  $du = (e^x + e^{-x}) dx$ . Then  $I = \int \frac{du}{u} = \ln|u| + C = \ln|e^x - e^{-x}| + C$ .

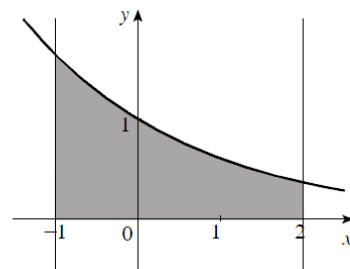
92.  $I = \int \frac{e^{-1/x}}{x^2} dx$ . Let  $u = -\frac{1}{x}$ , so  $du = \frac{dx}{x^2}$ . Then  $I = \int e^u du = e^u + C = e^{-1/x} + C$ .

$$94. \int_{-1}^0 \frac{1}{1 + e^{-2x}} dx = \int_{-1}^0 \frac{e^{2x}}{e^{2x} + 1} dx = \frac{1}{2} \ln(e^{2x} + 1) \Big|_{-1}^0 = \frac{1}{2} [\ln 2 - \ln(e^{-2} + 1)]$$

101. Let  $u = (\ln x)^2 \Rightarrow du = \frac{2 \ln x}{x} dx \Rightarrow \frac{\ln x}{x} dx = \frac{1}{2} du$ ,  $x = 1 \Rightarrow u = 0$ , and  $x = e \Rightarrow u = 1$ . Then

$$\int_1^e \frac{\ln x}{x} e^{(\ln x)^2} dx = \frac{1}{2} \int_0^1 e^u du = \frac{1}{2} e^u \Big|_0^1 = \frac{1}{2} (e - 1).$$

$$102. A = \int_{-1}^2 e^{-x/2} dx = -2e^{-x/2} \Big|_{-1}^2 \\ = -2(e^{-1} - e^{1/2}) = 2(\sqrt{e} - 1/e)$$



116. Differentiating the equation with respect to  $x$ , we have  $\frac{d}{dx} \int_0^y e^t dt + \frac{d}{dx} \int_0^x \cos t dt = 0$ . Using Part 1 of the Fundamental Theorem of Calculus and the Chain Rule, we have  $e^y \cdot \frac{d}{dx}(y) + \cos x = 0 \Leftrightarrow e^y \frac{dy}{dx} = -\cos x \Leftrightarrow \frac{dy}{dx} = -e^{-y} \cos x$ .

## 6.4 General Exponential and Logarithmic Functions

22.  $h(t) = 4^{t-1} \Rightarrow h'(t) = (\ln 4) 4^{t-1}$

24.  $f(u) = 2^{u^2} \Rightarrow f'(u) = \ln 2 (2^{u^2} \cdot 2u) = (2 \ln 2) u \cdot 2^{u^2}$

27.  $f(x) = x^e + e^x \Rightarrow f'(x) = ex^{e-1} + e^x$

39.  $y = (x+2)^{1/x} \Rightarrow \ln y = \ln (x+2)^{1/x} = \frac{1}{x} \ln(x+2) \Rightarrow \frac{y'}{y} = -\frac{1}{x^2} \ln(x+2) + \frac{1}{x} \cdot \frac{1}{x+2} \Rightarrow$   
 $y' = \left[ \frac{1}{x(x+2)} - \frac{\ln(x+2)}{x^2} \right] (x+2)^{1/x}$

49. Let  $u = 1 + 3^x$ , so  $du = 3^x \ln 3 dx$ . Then  $\int \frac{3^x}{1+3^x} dx = \frac{1}{\ln 3} \int \frac{du}{u} = \frac{\ln u}{\ln 3} + C = \frac{\ln(3^x + 1)}{\ln 3} + C$ .

50. Let  $u = \log x$ , so  $du = \frac{1}{\ln 10} \frac{dx}{x}$ . Then  $\int \frac{\sqrt{\log x}}{x} dx = \ln 10 \int u^{1/2} du = \frac{2 \ln 10}{3} u^{3/2} + C = \frac{2 \ln 10}{3} (\log x)^{3/2} + C$  or  
 $\frac{2 (\ln x)^{3/2}}{3 \sqrt{\ln 10}} + C$ .