6. 1

31. $g(u) = \ln \dfrac{u}{u+1} = \ln u - \ln(u+1) \Rightarrow g'(u) = \dfrac{1}{u} - \dfrac{1}{u+1} = \dfrac{1}{u(u+1)}$

43. $f'(x) = \dfrac{d}{dx}\left(x^2 \ln \cos x\right) = 2x \ln \cos x + x^2 \dfrac{d}{dx} \ln \cos x = 2x \ln \cos x + x^2 \cdot \dfrac{-\sin x}{\cos x} = 2x \ln \cos x - x^2 \tan x$

49. $\ln y - x \ln x = -1 \Rightarrow \dfrac{1}{y} \cdot y' - \left(\ln x + x \cdot \dfrac{1}{x}\right) = 0 \Rightarrow y' = y(\ln x + 1)$

56. $y - \ln\left(x^2 + y^2\right) = 0 \Rightarrow y' - \dfrac{2x + 2yy'}{x^2 + y^2} = 0$. Substituting $x = 1$ and $y = 0$ into the equation gives $y' - \dfrac{2+0}{1+0} = 0$ or $y' = 2$, the slope of the required tangent line. An equation is $y - 0 = 2(x - 1)$ or $y = 2x - 2$.

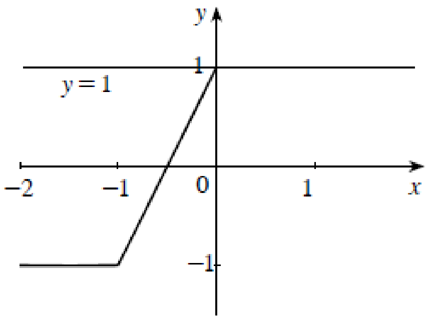
84. Let $u = \ln x$. Then $du = \dfrac{dx}{x}$, so $I = \displaystyle\int \frac{\ln x \sqrt{1 + \ln x}}{x}\, dx = \int u(1+u)^{1/2}\, du$. Now let $v = 1 + u$. Then $dv = du$ and

$I = \int (v-1)\, v^{1/2}\, dv = \int \left(v^{3/2} - v^{1/2}\right) dv = \frac{2}{5} v^{5/2} - \frac{2}{3} v^{3/2} + C = \frac{2}{15} v^{3/2}(3v - 5) + C$

$= \frac{2}{15}(1+u)^{3/2}(3u - 2) + C = \frac{2}{15}(\ln x + 1)^{3/2}(3 \ln x - 2) + C$

6.2

18. $f(x) = |x+1| - |x|$ is not one-to-one. The horizontal line $y = 1$ cuts the graph of f at infinitely many points.



21. By inspection $f(0) = -1$, so $f^{-1}(-1) = 0$.

24. By inspection $f(0) = 2$, so $f^{-1}(2) = 0$.

57. Observe that $f(2) = \int_2^2 \frac{dt}{\sqrt{1+t^3}} = 0$, showing that $(2, 0)$ lies on the graph of f. Next, observe that $f'(x) = \frac{d}{dx}\int_2^x \frac{dt}{\sqrt{1+t^3}} = \frac{1}{\sqrt{1+x^3}}$ (by the FTC, Part 1). Therefore,

$$\left(f^{-1}\right)'(0) = \frac{1}{f'\left(f^{-1}(0)\right)} = \frac{1}{f'(2)} = \sqrt{1+2^3} = 3.$$

60. a. $g'(x) = \frac{1}{f'(g(x))} = [f'(g(x))]^{-1}$, so by the Chain Rule,

$$g''(x) = -[f'(g(x))]^{-2}\frac{d}{dx}[f'(g(x))] = -[f'(g(x))]^{-2} f''(g(x))\, g'(x) = -\frac{f''(g(x))}{[f'(g(x))]^3}.$$

b. f is increasing on $(a, b) \Rightarrow f' > 0$ on (a, b). Also, the graph of f is concave upward on $(a, b) \Rightarrow f'' > 0$ on (a, b). Using the result of part a, we see that $g'' < 0$ on (a, b), and so the graph of g is concave downward on (a, b).