11.3 The Dot Product ET 10.3

42. $\mathbf{a} = \langle 1, 2, 0 \rangle$ and $\mathbf{b} = \langle -3, 0, -4 \rangle$.

a. $\operatorname{proj}_{\mathbf{a}} \mathbf{b} = \left(\dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|^2}\right) \mathbf{a} = \left(\dfrac{\langle 1, 2, 0 \rangle \cdot \langle -3, 0, -4 \rangle}{1+4}\right) \langle 1, 2, 0 \rangle = -\frac{3}{5} \langle 1, 2, 0 \rangle = \left\langle -\frac{3}{5}, -\frac{6}{5}, 0 \right\rangle$

b. $\operatorname{proj}_{\mathbf{b}} \mathbf{a} = \left(\dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|^2}\right) \mathbf{b} = \left(\dfrac{\langle 1, 2, 0 \rangle \cdot \langle -3, 0, -4 \rangle}{9+16}\right) \langle -3, 0, -4 \rangle = -\frac{3}{25} \langle -3, 0, -4 \rangle = \left\langle \frac{9}{25}, 0, \frac{12}{25} \right\rangle$

47. $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$. Because $\operatorname{proj}_{\mathbf{a}} \mathbf{b} = \left(\dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|^2}\right) \mathbf{a} = \left[\dfrac{(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) \cdot (2\mathbf{i} - \mathbf{j} + \mathbf{k})}{1+4+9}\right] (\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) = \left(\dfrac{2-2+3}{14}\right) (\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) = \frac{3}{14}\mathbf{i} + \frac{3}{7}\mathbf{j} + \frac{9}{14}\mathbf{k}$ is parallel to $\mathbf{a}$, we can write

$$\mathbf{b} = \operatorname{proj}_{\mathbf{a}} \mathbf{b} + (\mathbf{b} - \operatorname{proj}_{\mathbf{a}} \mathbf{b}) = \left(\tfrac{3}{14}\mathbf{i} + \tfrac{3}{7}\mathbf{j} + \tfrac{9}{14}\mathbf{k}\right) + \left[(2\mathbf{i} - \mathbf{j} + \mathbf{k}) - \left(\tfrac{3}{14}\mathbf{i} + \tfrac{3}{7}\mathbf{j} + \tfrac{9}{14}\mathbf{k}\right)\right]$$
$$= \left(\tfrac{3}{14}\mathbf{i} + \tfrac{3}{7}\mathbf{j} + \tfrac{9}{14}\mathbf{k}\right) + \left(\tfrac{25}{14}\mathbf{i} - \tfrac{10}{7}\mathbf{j} + \tfrac{5}{14}\mathbf{k}\right)$$

11.4 The Cross Product ET 10.4

13. One such vector is $\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -3 & 4 \\ -1 & 1 & -2 \end{vmatrix} = 2\mathbf{i} - \mathbf{k}$; another is $\mathbf{b} \times \mathbf{a} = -2\mathbf{i} + \mathbf{k}$.

17. For $P(1, 0, 0)$, $Q(0, 1, 0)$, and $R(0, 0, 1)$, $\overrightarrow{PQ} = -\mathbf{i} + \mathbf{j}$ and $\overrightarrow{PR} = -\mathbf{i} + \mathbf{k}$, so $\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, so the area of $\triangle PQR$ is $\frac{1}{2}\left|\overrightarrow{PQ} \times \overrightarrow{PR}\right| = \frac{1}{2}|\mathbf{i} + \mathbf{j} + \mathbf{k}| = \frac{1}{2}\sqrt{1+1+1} = \frac{\sqrt{3}}{2}$.

35. For $P(1, 0, 1)$, $Q(2, 3, 1)$, $R(-1, 2, -3)$, and $S\left(\frac{2}{3}, -1, 1\right)$, $\overrightarrow{PQ} = \langle 1, 3, 0 \rangle$, $\overrightarrow{PR} = \langle -2, 2, -4 \rangle$, and $\overrightarrow{PS} = \left\langle -\frac{1}{3}, -1, 0 \right\rangle$, so $\overrightarrow{PQ} \cdot \left(\overrightarrow{PR} \times \overrightarrow{PS}\right) = \begin{vmatrix} 1 & 3 & 0 \\ -2 & 2 & -4 \\ -\frac{1}{3} & -1 & 0 \end{vmatrix} = 1(0-4) - 3\left(0 - \frac{4}{3}\right) = 0$. Thus, the points are coplanar.

11.5 Lines and Planes in Space ET 10.5

12. The direction of the given line is the same as that of $\mathbf{v} = \langle 3, -3, 1 \rangle$, so parametric equations of the required line are $x = -1 + 3t$, $y = 3 - 3t$, $z = -2 + t$. To find the point where the line intersects the yz-plane, set $x = 0 \Rightarrow t = \frac{1}{3} \Rightarrow y = 2$ and $z = -\frac{5}{3}$. Thus, the required point is $\left(0, 2, -\frac{5}{3}\right)$.

27. A normal to the given plane is $\mathbf{n} = \langle 2, 3, -1 \rangle$. Because the required plane is parallel to the given plane, $\mathbf{n}$ is also normal to the required plane, so an equation is $2(x-3) + 3(y-6) - 1(z+2) = 0 \Leftrightarrow 2x + 3y - z = 26$.

43. A normal to the plane $x + y + 2z = 6$ is $\mathbf{n} = \langle 1, 1, 2 \rangle$ and a vector parallel to the line $L: x = 1 + t, y = 2 + t, z = -1 + t$ is $\mathbf{v} = \langle 1, 1, 1 \rangle$, so the angle between the normal to the plane and the line is $\theta = \cos^{-1}\left(\dfrac{|\mathbf{n} \cdot \mathbf{v}|}{|\mathbf{n}|\,|\mathbf{v}|}\right) = \cos^{-1}\left(\dfrac{|\langle 1, 1, 2 \rangle \cdot \langle 1, 1, 1 \rangle|}{\sqrt{1+1+4}\sqrt{1+1+1}}\right) = \cos^{-1}\dfrac{4}{\sqrt{6}\sqrt{3}} \approx 19.5^\circ$. Therefore, the required angle is about $90^\circ - 19.5^\circ = 70.5^\circ$.