13.2

3. Along $y = 0$, $\lim_{(x,y)\to(0,0)} \frac{3xy}{3x^2+y^2} = \lim_{x\to 0} \frac{0}{3x^2} = \lim_{x\to 0} = 0$. Along $y = x$,

$\lim_{(x,y)\to(0,0)} \frac{3xy}{3x^2+y^2} = \lim_{x\to 0} \frac{3x^2}{3x^2+x^2} = \lim_{x\to 0} \frac{3}{4} = \frac{3}{4}$. Because these two limits are not equal, the given limit does not exist.

4. Along $y = 0$, $\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4} = \lim_{x\to 0} \frac{0}{x^2} = \lim_{x\to 0} 0 = 0$. Along $x = y^2$,

$\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4} = \lim_{y\to 0} \frac{y^4}{2y^4} = \lim_{y\to 0} \frac{1}{2} = \frac{1}{2}$. Because these two limits are not equal, the given limit does not exist.

15. $\lim_{(x,y)\to(1,2)} \frac{2x^2-3y^3+4}{3-xy} = \frac{2(1)^2-3(2)^3+4}{3-(1)(2)} = -18$

19. $\lim_{(x,y)\to(0^+,0^+)} \frac{e^{\sqrt{x+y}}}{x+y-1} = \frac{e^0}{-1} = -1$

23. $\lim_{(x,y)\to(2,1)} \ln\left(x^2-3y\right) = \ln(4-3) = \ln 1 = 0$

31. The function f is a rational function and is continuous at all points where its denominator is nonzero. Thus, f is continuous on $\{(x, y) \mid 2x + 3y \neq 1\}$.

33. The function g is continuous for all (x, y) such that $x + y \geq 0$ and $x - y \geq 0$; that is, on $\{(x, y) \mid x \geq 0, |y| \leq x\}$.

35. We require that $x \geq 0$ and $y \neq 0$. Thus, F is continuous on $\{(x, y) \mid x \geq 0, y \neq 0\}$.

13.3

10. $f_x(x,y) = \frac{\partial}{\partial x}\left(2x^2 - y^3\right)^4 = 4\left(2x^2 - y^3\right)^3 \frac{\partial}{\partial x}\left(2x^2 - y^3\right) = 4\left(2x^2 - y^3\right)^3 (4x) = 16x\left(2x^2 - y^3\right)^3$ and

$f_y(x,y) = \frac{\partial}{\partial y}\left(2x^2 - y^3\right)^4 = 4\left(2x^2 - y^3\right)^3 \frac{\partial}{\partial y}\left(2x^2 - y^3\right) = 4\left(2x^2 - y^3\right)^3\left(-3y^2\right) = -12y^2\left(2x^2 - y^3\right)^3$.

12. $h_u(u,v) = \dfrac{\partial}{\partial u}\ln\left(u^2 + v^2\right) = \dfrac{\frac{\partial}{\partial u}\left(u^2 + v^2\right)}{u^2 + v^2} = \dfrac{2u}{u^2 + v^2}$ and by symmetry, $h_v(u,v) = \dfrac{2v}{u^2 + v^2}$.

14. $f_x(x,y) = \frac{\partial}{\partial x}\left(e^x \cos y + e^y \sin x\right) = e^x \cos y + e^y \cos x$ and $f_y(x,y) = \frac{\partial}{\partial y}\left(e^x \cos y + e^y \sin x\right) = -e^x \sin y + e^y \sin x$.

20. $\dfrac{\partial z}{\partial x} = \dfrac{\partial}{\partial x}\left[\ln\left(e^x + y^2\right)\right] = \dfrac{\frac{\partial}{\partial x}\left(e^x + y^2\right)}{e^x + y^2} = \dfrac{e^x}{e^x + y^2}$ and $\dfrac{\partial z}{\partial y} = \dfrac{\partial}{\partial y}\left[\ln\left(e^x + y^2\right)\right] = \dfrac{\frac{\partial}{\partial y}\left(e^x + y^2\right)}{e^x + y^2} = \dfrac{2y}{e^x + y^2}$.

21. $f_x(x,y) = \frac{\partial}{\partial x} y^x = y^x \ln y$ and $f_y(x,y) = \frac{\partial}{\partial y} y^x = xy^{x-1}$.

22. $f_x(x,y) = \frac{\partial}{\partial x}\int_x^y \cos t\,dt = -\frac{\partial}{\partial x}\int_y^x \cos t\,dt = -\cos x$ and $f_y(x,y) = \frac{\partial}{\partial y}\int_x^y \cos t\,dt = \cos y$.

23. $f_x(x,y) = \frac{\partial}{\partial x}\int_x^y te^{-t}\,dt = -\frac{\partial}{\partial x}\int_y^x te^{-t}\,dt = -xe^{-x}$ and $f_y(x,y) = \frac{\partial}{\partial y}\int_x^y te^{-t}\,dt = ye^{-y}$.

31. $\dfrac{\partial}{\partial x}\left(xe^y + ye^{-x} + e^z\right) = \dfrac{\partial}{\partial x}(10) \Rightarrow e^y - ye^{-x} + e^z\dfrac{\partial z}{\partial x} = 0 \Rightarrow \dfrac{\partial z}{\partial x} = \dfrac{ye^{-x} - e^y}{e^z}$ and $\dfrac{\partial}{\partial y}\left(xe^y + ye^{-x} + e^z\right) = \dfrac{\partial}{\partial y}(10)$

$\Rightarrow xe^y + e^{-x} + e^z\dfrac{\partial z}{\partial y} = 0 \Rightarrow \dfrac{\partial z}{\partial y} = -\dfrac{xe^y + e^{-x}}{e^z}$.

33. $\dfrac{\partial}{\partial x}\left[\ln\left(x^2 + z^2\right) + yz^3 + 2x^2\right] = \dfrac{\partial}{\partial x}(10) \Rightarrow \dfrac{2x + 2z\frac{\partial z}{\partial x}}{x^2 + z^2} + 3yz^2\dfrac{\partial z}{\partial x} + 4x = 0 \Rightarrow \dfrac{\partial z}{\partial x} = -\dfrac{2x\left(2x^2 + 2z^2 + 1\right)}{z\left(3yz^3 + 3x^2yz + 2\right)}$ and

$\dfrac{\partial}{\partial y}\left[\ln\left(x^2 + z^2\right) + yz^3 + 2x^2\right] = \dfrac{\partial}{\partial y}(10) \Rightarrow \dfrac{2z\frac{\partial z}{\partial y}}{x^2 + z^2} + z^3 + 3yz^2\dfrac{\partial z}{\partial y} = 0 \Rightarrow \dfrac{\partial z}{\partial y} = -\dfrac{z^2\left(x^2 + z^2\right)}{3yz^3 + 3x^2yz + 2}$.

35. $g_x(x,y) = \frac{\partial}{\partial x}\left(x^3y^2 + xy^3 - 2x + 3y + 1\right) = 3x^2y^2 + y^3 - 2$,

$g_y(x,y) = \frac{\partial}{\partial y}\left(x^3y^2 + xy^3 - 2x + 3y + 1\right) = 2x^3y + 3xy^2 + 3$, $g_{xx}(x,y) = \frac{\partial}{\partial x}\left(3x^2y^2 + y^3 - 2\right) = 6xy^2$,

$g_{yy} = \frac{\partial}{\partial y}\left(2x^3y + 3xy^2 + 3\right) = 2x^3 + 6xy$, $g_{xy} = \frac{\partial}{\partial y}\left(3x^2y^2 + y^3 - 2\right) = 6x^2y + 3y^2$, and

$g_{yx} = \frac{\partial}{\partial x}\left(2x^3y + 3xy^2 + 3\right) = 6x^2y + 3y^2$.

37. $\dfrac{\partial w}{\partial u} = \dfrac{\partial}{\partial u}\left[\cos(2u - v) + \sin(2u + v)\right] = -2\sin(2u - v) + 2\cos(2u + v)$,

$\dfrac{\partial w}{\partial v} = \dfrac{\partial}{\partial v}\left[\cos(2u - v) + \sin(2u + v)\right] = \sin(2u - v) + \cos(2u + v)$, $\dfrac{\partial^2 w}{\partial u^2} = -4\cos(2u - v) - 4\sin(2u + v)$,

$\dfrac{\partial^2 w}{\partial v^2} = -\cos(2u - v) - \sin(2u + v)$, and $\dfrac{\partial^2 w}{\partial u\,\partial v} = \dfrac{\partial^2 w}{\partial v\,\partial u} = 2\cos(2u - v) - 2\sin(2u + v)$.

47. $f_x(x,y) = \frac{\partial}{\partial x}\left(x\sin^2 y + y^2\cos x\right) = \sin^2 y - y^2 \sin x \Rightarrow f_{xy}(x,y) = \frac{\partial}{\partial y}\left(\sin^2 y - y^2 \sin x\right) = 2\sin y\cos y - 2y\sin x$ and $f_y(x,y) = \frac{\partial}{\partial y}\left(x\sin^2 y + y^2\cos x\right) = 2x\sin y\cos y + 2y\cos x \Rightarrow$ $f_{yx}(x,y) = \frac{\partial}{\partial x}(2x\sin y\cos y + 2y\cos x) = 2\sin y\cos y - 2y\sin x$, so $f_{xy} = f_{yx}$.

49. $f_x(x,y) = \dfrac{\partial}{\partial x}\left[\tan^{-1}\left(x^2+y^3\right)\right] = \dfrac{2x}{1+(x^2+y^3)^2} \Rightarrow$

$$f_{xy}(x,y) = \frac{\partial}{\partial y}\left\{2x\left[1+\left(x^2+y^3\right)^2\right]^{-1}\right\} = 2x(-1)\left[1+\left(x^2+y^3\right)^2\right]^{-2}(2)\left(x^2+y^3\right)\left(3y^2\right) = -\frac{12xy^2\left(x^2+y^3\right)}{\left[1+(x^2+y^3)^2\right]^2}$$

and $f_y(x,y) = \dfrac{\partial}{\partial y}\left[\tan^{-1}\left(x^2+y^3\right)\right] = \dfrac{3y^2}{1+(x^2+y^3)^2} \Rightarrow$

$$f_{yx}(x,y) = \frac{\partial}{\partial x}\left\{3y^2\left[1+\left(x^2+y^3\right)^2\right]^{-1}\right\} = 3y^2(-1)\left[1+\left(x^2+y^3\right)^2\right]^{-2}(2)\left(x^2+y^3\right)(2x) = -\frac{12xy^2\left(x^2+y^3\right)}{\left[1+(x^2+y^3)^2\right]^2},$$

so $f_{xy} = f_{yx}$.