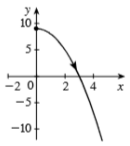
10.2 Plane Curves and Parametric Equations

3. a. $\left.\begin{array}{l} x = \sqrt{t} \\ y = 9 - t \end{array}\right\} \Rightarrow y = 9 - x^2, x \geq 0$

b.

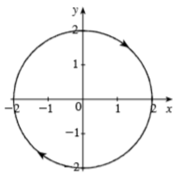


The orientation is found by observing that as t increases, so does x.

9. a. $\left.\begin{array}{l} x = 2\sin\theta \\ y = 2\cos\theta \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \sin\theta = \frac{1}{2}x \\ \cos\theta = \frac{1}{2}y \end{array}\right\} \Rightarrow$

$\left(\frac{1}{2}x\right)^2 + \left(\frac{1}{2}y\right)^2 = \sin^2\theta + \cos^2\theta = 1$, so $x^2 + y^2 = 4$.

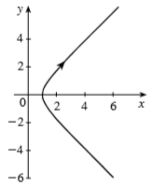
b.



Observe that as θ increases from 0 to 2π, the curve C is traced once in a clockwise direction starting from the point $(0, 2)$.

17. a. $\left.\begin{array}{l} x = \sec\theta \\ y = \tan\theta \end{array}\right\} \Rightarrow \left\{\begin{array}{l} x^2 = \sec^2\theta \\ y^2 = \tan^2\theta \end{array}\right\}$ From the identity $\sec^2\theta = 1 + \tan^2\theta$, we obtain $x^2 - y^2 = 1$, $x \geq 1$.

b.



37. a. $x = x_1 + (x_2 - x_1)t$ and $y = y_1 + (y_2 - y_1)t$. From the first equation, we find $t = \dfrac{x - x_1}{x_2 - x_1}$. Substituting this expression into the second equation gives $y = y_1 + (y_2 - y_1)\left(\dfrac{x - x_1}{x_2 - x_1}\right) = y_1 + \dfrac{y_2 - y_1}{x_2 - x_1}x - \dfrac{x_1(y_2 - y_1)}{x_2 - x_1}$, which is a linear equation in x and y. Since (x_1, y_1) and (x_2, y_2) both satisfy the equation, we see that $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ both lie on the line.

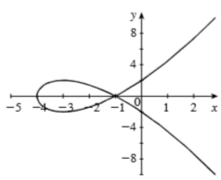
b. If $t = 0$, $x = x_1$ and $y = y_1$, so (x_1, y_1) is on the line. If $t = 1$, then $x = x_2$ and $y = y_2$, so (x_2, y_2) is on the line. As t increases from $t = 0$ to $t = 1$, the line segment joining $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is traced out.

10.3 The Calculus of Parametric Equations

4. $x = e^{2t}$, $y = \ln t \Rightarrow \frac{dx}{dt} = 2e^{2t}$ and $\frac{dy}{dt} = 1/t$. The slope of the tangent line at $t = 1$ is $\left.\dfrac{dy}{dx}\right|_{t=1} = \left.\dfrac{dy/dt}{dx/dt}\right|_{t=1} = \left.\dfrac{1/t}{2e^{2t}}\right|_{t=1} = \dfrac{1}{2e^2}$.

6. $x = 2(\theta - \sin\theta)$, $y = 2(1 - \cos\theta) \Rightarrow \frac{dx}{d\theta} = 2(1 - \cos\theta)$ and $\frac{dy}{d\theta} = 2\sin\theta$. The slope of the tangent line at $\theta = \frac{\pi}{6}$ is $\left.\dfrac{dy}{dx}\right|_{\theta=\pi/6} = \left.\dfrac{dy/d\theta}{dx/d\theta}\right|_{\theta=\pi/6} = \left.\dfrac{2\sin\theta}{2(1 - \cos\theta)}\right|_{\theta=\pi/6} = \dfrac{1}{2\left(1 - \frac{\sqrt{3}}{2}\right)} = \dfrac{1}{2 - \sqrt{3}} = 2 + \sqrt{3}$.

13. $x = t^2 - 4$, $y = t^3 - 3t \Rightarrow \frac{dx}{dt} = 2t$ and $\frac{dy}{dt} = 3t^2 - 3$. To find the point(s) where the tangent line is horizontal, set $\frac{dy}{dt} = 0 \Rightarrow 3t^2 - 3 = 3(t + 1)(t - 1) = 0 \Rightarrow t = \pm 1$. Since $\frac{dx}{dt} \neq 0$ at either of these t-values, the required points are $(x(-1), y(-1)) = (-3, 2)$ and $(x(1), y(1)) = (-3, -2)$. To find the point(s) where the tangent line is vertical, set $\frac{dx}{dt} = 0 \Rightarrow 2t = 0 \Rightarrow t = 0$. Since $\frac{dy}{dt} \neq 0$ at this value of t, we see that the required point is $(x(0), y(0)) = (-4, 0)$.



19. $x = \sqrt{t}$, $y = \dfrac{1}{t} \Rightarrow \dfrac{dx}{dt} = \dfrac{1}{2\sqrt{t}}$ and $\dfrac{dy}{dt} = -\dfrac{1}{t^2}$, so $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{-1/t^2}{1/\left(2t^{1/2}\right)} = -2t^{-3/2} = -\dfrac{2}{t^{3/2}}$ and

$\dfrac{d^2y}{dx^2} = \dfrac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \dfrac{\frac{d}{dt}\left(-2t^{-3/2}\right)}{1/\left(2t^{1/2}\right)} = 3t^{-5/2} \cdot 2t^{1/2} = \dfrac{6}{t^2}$.