2.3 The Product and Quotient Rules

33. $f'(x) = (2x-1)\frac{d}{dx}\left(x^2+3\right) + \left(x^2+3\right)\frac{d}{dx}(2x-1) = (2x-1)(2x) + \left(x^2+3\right)(2) \Rightarrow$
$f'(1) = (2-1)(2) + (4)(2) = 10$

34. $f'(x) = \dfrac{(2x-1)\frac{d}{dx}(2x+1) - (2x+1)\frac{d}{dx}(2x-1)}{(2x-1)^2} = \dfrac{(2x-1)(2) - (2x+1)(2)}{(2x-1)^2} \Rightarrow$
$f'(2) = \dfrac{[2(2)-1](2) - [2(2)+1](2)}{[2(2)-1]^2} = \dfrac{6-10}{9} = -\dfrac{4}{9}$

56. $f(x) = \dfrac{x+1}{x-1} \Rightarrow f'(x) = \dfrac{(x-1)(1)-(x+1)(1)}{(x-1)^2} = -\dfrac{2}{(x-1)^2} \Rightarrow$
$f''(x) = -\left[\dfrac{(x-1)^2(0) - 2\frac{d}{dx}\left(x^2-2x+1\right)}{(x-1)^4}\right] = \dfrac{2(2x-2)}{(x-1)^4} = \dfrac{4}{(x-1)^3}$

62. a. $f(x) = 8x^7 - 6x^5 + 4x^3 - x \Rightarrow f'(x) = 56x^6 - 30x^4 + 12x^2 - 1 \quad \Rightarrow \quad f''(x) = 336x^5 - 120x^3 + 24x \Rightarrow$
$f'''(x) = 1680x^4 - 360x^2 + 24$, so $f'''(0) = 24$.

b. $y = x^{-1} \Rightarrow y' = -x^{-2} \quad \Rightarrow \quad y'' = 2x^{-3} \Rightarrow y''' = -6x^{-4}$, so $y'''|_{x=1} = -6(1)^{-4} = -6$.

2.5 Derivatives of Trigonometric Functions

18. $y' = \dfrac{d}{dx}\left(\dfrac{\sin x\cos x}{1+\csc x}\right) = \dfrac{(1+\csc x)[(\sin x)(-\sin x) + \cos x(\cos x)] - \sin x\cos x(-\csc x\cot x)}{(1+\csc x)^2}$
$= \dfrac{(1+\csc x)\cos 2x + \cos x\cot x}{(1+\csc x)^2}$ (since $\cos^2 x - \sin^2 x = \cos 2x$)

20. $s' = \dfrac{d}{dt}\left(\dfrac{1-\tan t}{1+\cot t}\right) = \dfrac{(1+\cot t)\frac{d}{dt}(1-\tan t) - (1-\tan t)\frac{d}{dt}(1+\cot t)}{(1+\cot t)^2}$
$= \dfrac{(1+\cot t)\left(-\sec^2 t\right) - (1-\tan t)\left(-\csc^2 t\right)}{(1+\cot t)^2} = \dfrac{\left(1+\frac{\cos t}{\sin t}\right)\left(-\frac{1}{\cos^2 t}\right) - \left(1-\frac{\sin t}{\cos t}\right)\left(-\frac{1}{\sin^2 t}\right)}{\left(1+\frac{\cos t}{\sin t}\right)^2}$
$= \dfrac{2\cos^2 t - 2\sin t\cos t - 1}{\cos^2 t(\cos t + \sin t)^2}$

34. $y' = \dfrac{d}{dx}(\csc x - 2\cos x) = -\csc x\cot x + 2\sin x \Rightarrow y'|_{x=\pi/6} = (-2)\sqrt{3} + 2\left(\frac{1}{2}\right) = 1 - 2\sqrt{3}$

38. $g'(x) = \dfrac{d}{dx}(x + \sin x) = 1 + \cos x$. Setting $1 + \cos x = 1$ gives $\cos x = 0 \Rightarrow x = \dfrac{(2k+1)\pi}{2}$, $k = 0, \pm 1, \pm 2, \ldots$.