13.2 Limits and Continuity ET 12.2

5. Along $y = 0$, $\lim\limits_{(x,y)\to(0,0)} \dfrac{2xy}{\sqrt{x^4+y^4}} = \lim\limits_{x\to 0} \dfrac{0}{\sqrt{x^4}} = \lim\limits_{x\to 0} 0 = 0$. Along $y = x$, $\lim\limits_{(x,y)\to(0,0)} \dfrac{2xy}{\sqrt{x^4+y^4}} = \lim\limits_{x\to 0} \dfrac{2x^2}{\sqrt{x^4+x^4}} = \lim\limits_{x\to 0} \sqrt{2} = \sqrt{2}$. Because these two limits are not equal, the given limit does not exist.

9. Along the x-axis, $\lim\limits_{(x,y,z)\to(0,0,0)} \dfrac{xy+yz+xz}{x^2+y^2+z^2} = \lim\limits_{x\to 0} \dfrac{0}{x^2} = 0$. Let C denote the curve with parametric equations $x = t$, $y = t$, $z = t$. Then along C, $\lim\limits_{(x,y,z)\to(0,0,0)} \dfrac{xy+yz+xz}{x^2+y^2+z^2} = \lim\limits_{t\to 0} \dfrac{3t^2}{3t^2} = \lim\limits_{t\to 0} 1 = 1$. Because these two limits are not equal, the given limit does not exist.

15. $\lim\limits_{(x,y)\to(1,2)} \dfrac{2x^2-3y^3+4}{3-xy} = \dfrac{2(1)^2-3(2)^3+4}{3-(1)(2)} = -18$

19. $\lim\limits_{(x,y)\to(0^+,0^+)} \dfrac{e^{\sqrt{x+y}}}{x+y-1} = \dfrac{e^0}{-1} = -1$

13.3 Partial Derivatives ET 12.3

17. $g_u(u,v) = \dfrac{\partial}{\partial u}\left(\dfrac{uv}{u^2+v^3}\right) = \dfrac{\left(u^2+v^3\right)(v)-(uv)(2u)}{\left(u^2+v^3\right)^2} = \dfrac{v\left(v^3-u^2\right)}{\left(u^2+v^3\right)^2}$ and

$g_v(u,v) = \dfrac{\partial}{\partial v}\left(\dfrac{uv}{u^2+v^3}\right) = \dfrac{\left(u^2+v^3\right)(u)-(uv)\left(3v^2\right)}{\left(u^2+v^3\right)^2} = \dfrac{u\left(u^2-2v^3\right)}{\left(u^2+v^3\right)^2}$.

26. $f_u(u,v,w) = \frac{\partial}{\partial u}\left(ue^v - ve^u + we^u\right) = e^v - ve^u + we^u$, $f_v(u,v,w) = \frac{\partial}{\partial v}\left(ue^v - ve^u + we^u\right) = ue^v - e^u$, and $f_w(u,v,w) = \frac{\partial}{\partial w}\left(ue^v - ve^u + we^u\right) = e^u$.

28. $\dfrac{\partial u}{\partial x} = \dfrac{\partial}{\partial x} x \sin\dfrac{y}{x+z} = \sin\dfrac{y}{x+z} + \left(x\cos\dfrac{y}{x+z}\right)\dfrac{\partial}{\partial x}\left[y(x+z)^{-1}\right] = \sin\dfrac{y}{x+z} + \left(x\cos\dfrac{y}{x+z}\right)\left[-y(x+z)^{-2}\right]$

$= \sin\dfrac{y}{x+z} - \dfrac{xy}{(x+z)^2}\cos\dfrac{y}{x+z}$,

$\dfrac{\partial u}{\partial y} = \dfrac{\partial}{\partial y} x \sin\dfrac{y}{x+z} = \left(x\cos\dfrac{y}{x+z}\right)\dfrac{\partial}{\partial y}\left(\dfrac{y}{x+z}\right) = \dfrac{x}{x+z}\cos\dfrac{y}{x+z}$, and

$\dfrac{\partial u}{\partial z} = \dfrac{\partial}{\partial z} x \sin\dfrac{y}{x+z} = x\cos\dfrac{y}{x+z}\dfrac{\partial}{\partial z}\left[y(x+z)^{-1}\right] = \left(x\cos\dfrac{y}{x+z}\right)\left[-y(x+z)^{-2}\right]$

$= -\dfrac{xy}{(x+z)^2}\cos\dfrac{y}{x+z}$

68. $\dfrac{\partial u}{\partial x} = \dfrac{\partial}{\partial x}\left(20x^2\cos\dfrac{y}{x}\right) = 40x\cos\dfrac{y}{x} + 20x^2\left(-\sin\dfrac{y}{x}\right)\left(-\dfrac{y}{x^2}\right) = 20\left(2x\cos\dfrac{y}{x} + y\sin\dfrac{y}{x}\right)$

and $\dfrac{\partial u}{\partial y} = \dfrac{\partial}{\partial y} 20x^2\cos\dfrac{y}{x} = 20x^2\left(-\sin\dfrac{y}{x}\right)\left(\dfrac{1}{x}\right) = -20x\sin\dfrac{y}{x}$. Therefore,

$x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 40x^2\cos\dfrac{y}{x} + 20xy\sin\dfrac{y}{x} - 20xy\sin\dfrac{y}{x} = 40x^2\cos\dfrac{y}{x} = 2u$, as was to be shown.