聯合微積分第一次作業解答

1.1

7. Use the graph of the function f to determine whether each statement is true or false.

True a. $\lim_{x\to -3^+} f(x) = 2$
True b. $\lim_{x\to 0} f(x) = 2$
False c. $\lim_{x\to 2} f(x) = 1$
True d. $\lim_{x\to 4^-} f(x) = 3$
True e. $\lim_{x\to 4^+} f(x)$ does not exist.
False f. $\lim_{x\to 4} f(x) = 2$

8. Use the graph of the function f to determine whether each statement is true or false.

a. $\lim_{x\to -3^+} f(x) = 1$ ans. True
b. $\lim_{x\to 0} f(x) = f(0)$ ans. True
c. $\lim_{x\to 2^-} f(x) = 2$ ans. True
d. $\lim_{x\to 2^+} f(x) = 3$ ans. True
e. $\lim_{x\to 3} f(x)$ does not exist. ans. False
f. $\lim_{x\to 5^-} f(x) = 3$ ans. True

33. The Heaviside Function

A generalization of the unit step function or Heaviside function H of Example 3 is the function H_c defined by

$$H_c(t-t_0) = \begin{cases} 0, & \text{if } t < t_0 \\ c, & \text{if } t \geq t_0 \end{cases}$$

where c is a constant and $t_0 \geq 0$
show that if $c \neq 0$ then $\lim_{t\to t_0} H_c(t-t_0)$ does not exist.

Clearly,

$$\lim_{t\to t_0^+} H_c(t-t_0) = c$$

and

$$\lim_{t\to t_0^-} H_c(t-t_0) = 0$$

If $c \neq 0$, then

$$lim_{t\to t_0^+} H_c(t-t_0) \neq lim_{t\to t_0^-} H_c(t-t_0)$$

hence $\lim_{t\to t_0} H_c(t-t_0)$ does not exist.

34. The Square-Wave Function

The square-wave function f can be expressed in terms of the Heaviside function (exercise 33) as follow:

$$f(t) = H_k(t) - H_k(t-k) + H_k(t-2k) - H_k(t-3k) + H_k(t-4k) - ...$$

Referring to the following figure,
Show that $\lim_{t\to nk} f(t) = 1$ does not exist for n=1,2,3,...

when n=1,3,5,...

$$\lim_{t\to nk^+} f(t) = 0$$

and

$$\lim_{t\to nk^-} f(t) = k$$

when n=2,4,6,...

$$\lim_{t\to nk^+} f(t) = k$$

and

$$\lim_{t\to nk^-} f(t) = 0$$

so $\lim_{t\to nk} f(t)$ does not exist for n=1,2,3,...

1.2

26. You are given that $\lim_{x\to a} f(x) = 2$,$\lim_{x\to a} g(x) = 4$,and $\lim_{x\to a} h(x) = -1$,Find the indicated limit.

$$\lim_{x\to a} \frac{f(x)g(x)}{\sqrt{g(x)+5}} = \frac{\lim_{x\to a} f(x)g(x)}{\lim_{x\to a}\sqrt{g(x)+5}} = \frac{\lim_{x\to a} f(x)g(x)}{\sqrt{\lim_{x\to a} g(x)+5}} = \frac{2\times 4}{\sqrt{4+5}} = \frac{8}{3}$$

46. Find the limit ,if it exists.

$$\lim_{x\to 2} \frac{x^2-x-2}{x-2} = \lim_{x\to 2} \frac{(x-2)(x+1)}{x-2} = \lim_{x\to 2}(x+1) = 2+1 = 3$$

72. Find the limit ,if it exists.

$$\lim_{x\to 0^+} \frac{x}{1-\cos^2 x} = \lim_{x\to 0^+}\left(\frac{x}{1-\cos x}\cdot\frac{1}{1+\cos x}\right) = \lim_{x\to 0^+} \frac{\frac{1}{1+\cos x}}{\frac{1-\cos x}{x}} = \infty$$

not exist

91.Let

$$f(x) = \begin{cases} x^2, & \text{if x is rational} \\ -x^2, & \text{if x is irrational} \end{cases}$$

Show that $\lim_{x\to 0} f(x) = 0$.

Let $g(x) = x^2$ and $h(x) = -x^2$

Clearly,

$$h(x) \leq f(x) \leq g(x)$$

$\lim_{x\to 0} h(x) = 0 = \lim_{x\to 0} g(x)$ by squeeze theorem $\lim_{x\to 0} f(x) = 0$
92. The Dirichlet Function

$$f(x) = \begin{cases} 1, & \text{if x is rational} \\ 0, & \text{if x is irrational} \end{cases}$$

Show that for every $\boldsymbol{a}$ $\lim_{x\to a} f(x)$ does not exist.

任意實數附近不管要求有多近都有有理數點和無理數點，所以 $\lim_{x\to a} f(x)$ 不存在