9.1 Sequences

17. $\lim_{n\to\infty}\left(\frac{n-1}{n}-\frac{2n+1}{n^2}\right)=\lim_{n\to\infty}\frac{n^2-3n-1}{n^2}=\lim_{n\to\infty}\frac{1-\frac{3}{n}-\frac{1}{n^2}}{1}=1$

19. $\lim_{n\to\infty}\frac{2n^2-3n+4}{3n^2+1}=\lim_{n\to\infty}\frac{2-\frac{3}{n}+\frac{4}{n^2}}{3+\frac{1}{n^2}}=\frac{2}{3}$

29. $\lim_{n\to\infty}\frac{\sin\sqrt{n}}{\sqrt{n}}=0$ by the Squeeze Theorem: $-\frac{1}{\sqrt{n}}<\frac{\sin\sqrt{n}}{\sqrt{n}}<\frac{1}{\sqrt{n}}$ and $\lim_{n\to\infty}\left(\pm\frac{1}{\sqrt{n}}\right)=0$, so the sequence converges to 0.

49. Put $f(x)=\dfrac{1-\left(1-\frac{1}{x}\right)^9}{1-\left(1-\frac{1}{x}\right)}$. Then

$\lim_{x\to\infty}f(x)=\lim_{x\to\infty}\dfrac{1-\left(1-\frac{1}{x}\right)^9}{1-\left(1-\frac{1}{x}\right)}=\lim_{x\to\infty}\dfrac{-9\left(1-\frac{1}{x}\right)^8\left(\frac{1}{x^2}\right)}{-\frac{1}{x^2}}$ (by l'Hôpital's Rule) $=\lim_{x\to\infty}9\left(1-\frac{1}{x}\right)^8=9$, so by Theorem 1 we have $\lim_{x\to\infty}\dfrac{1-\left(1-\frac{1}{n}\right)^9}{1-\left(1-\frac{1}{n}\right)}=9$.

9.2 Series

11. $\sum_{n=0}^{\infty}2\left(-\frac{1}{\sqrt{2}}\right)^n=\frac{2}{1-\left(-\frac{1}{\sqrt{2}}\right)}=\frac{2}{1+\frac{1}{\sqrt{2}}}=\frac{2\sqrt{2}}{\sqrt{2}+1}\cdot\frac{\sqrt{2}-1}{\sqrt{2}-1}=2\sqrt{2}\left(\sqrt{2}-1\right)=2\left(2-\sqrt{2}\right)$

17. Since $\lim_{n\to\infty}\frac{2n}{3n+1}=\frac{2}{3}\neq 0$, the series diverges.

36. $\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{2n^2+n+1}{3n^2+2}=\lim_{n\to\infty}\frac{2+\frac{1}{n}+\frac{1}{n^2}}{3+\frac{2}{n^2}}=\frac{2}{3}\neq 0$, so $\sum_{n=0}^{\infty}\frac{2n^2+n+1}{3n^2+2}$ diverges.

38. $\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{3^n-1}{3^{n+1}}=\lim_{n\to\infty}\left(\frac{1}{3}-\frac{1}{3^{n+1}}\right)=\frac{1}{3}\neq 0$, so $\sum_{n=1}^{\infty}\frac{3^n-1}{3^{n+1}}$ diverges.