13.5 The Chain Rule ET 12.5

3. $\dfrac{dw}{dt} = \dfrac{\partial w}{\partial r}\dfrac{dr}{dt} + \dfrac{\partial w}{\partial s}\dfrac{ds}{dt} = (\cos s + s\cos r)\left(-2e^{-2t}\right) + (-r\sin s + \sin r)\left(3t^2 - 2\right)$
$= -2(\cos s + s\cos r)\,e^{-2t} + (\sin r - r\sin s)\left(3t^2 - 2\right)$

9. $\dfrac{\partial w}{\partial u} = \dfrac{\partial w}{\partial x}\dfrac{\partial x}{\partial u} + \dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial u} = \left(3x^2\right)(2u) + \left(3y^2\right)(2v) = 6\left(x^2u + y^2v\right)$ and
$\dfrac{\partial w}{\partial v} = \dfrac{\partial w}{\partial x}\dfrac{\partial x}{\partial v} + \dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial v} = \left(3x^2\right)(2v) + \left(3y^2\right)(2u) = 6\left(x^2v + y^2u\right)$

21. $\dfrac{du}{dt} = \dfrac{\partial u}{\partial x}\dfrac{dx}{dt} + \dfrac{\partial u}{\partial y}\dfrac{dy}{dt} = \dfrac{y^2 - x^2}{\left(x^2 + y^2\right)^2}\cdot 2\sec 2t\tan 2t - \dfrac{2xy}{\left(x^2 + y^2\right)^2}\cdot\sec^2 t$. If $t = 0$, then $x = 1$ and $y = 0$, so
$\left.\dfrac{du}{dt}\right|_{t=0} = \dfrac{-1}{1}\cdot 0 - 0 = 0.$

37. We write $F(x, y) = x^3 + y^3 - 3axy = 0$, so $\dfrac{dy}{dx} = -\dfrac{F_x}{F_y} = -\dfrac{3x^2 - 3ay}{3y^2 - 3ax} = -\dfrac{x^2 - ay}{y^2 - ax}$.

13.6 Directional Derivatives and Gradient Vectors ET 12.6

4. Here $\mathbf{u} = \cos\left(-\frac{\pi}{4}\right)\mathbf{i} + \sin\left(-\frac{\pi}{4}\right)\mathbf{j} = \frac{\sqrt{2}}{2}\mathbf{i} - \frac{\sqrt{2}}{2}\mathbf{j}$, $D_{\mathbf{u}}f(x, y) = y\cos xy\left(\frac{\sqrt{2}}{2}\right) + x\cos xy\left(-\frac{\sqrt{2}}{2}\right)$, and
$D_{\mathbf{u}}f(1, 0) = 0 + 1\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\sqrt{2}}{2}$.

9. $f_x(x, y, z) = \frac{\partial}{\partial x}\left(xe^{yz}\right) = e^{yz}$, $f_y(x, y, z) = xze^{yz}$, and $f_z(x, y, z) = xye^{yz}$, so
$\nabla f(1, 0, 2) = \left.\left(e^{yz}\mathbf{i} + xze^{yz}\mathbf{j} + xye^{yz}\mathbf{k}\right)\right|_{(1,0,2)} = \mathbf{i} + 2\mathbf{j}$.

15. Here $\mathbf{u} = \dfrac{\mathbf{v}}{|\mathbf{v}|} = \dfrac{-\mathbf{i} + 3\mathbf{j}}{\sqrt{(-1)^2 + 3^2}} = -\frac{\sqrt{10}}{10}\mathbf{i} + \frac{3\sqrt{10}}{10}\mathbf{j}$, $f_x(x, y) = -\dfrac{2y}{(x - y)^2}$, and $f_y(x, y) = \dfrac{2x}{(x - y)^2}$, so
$D_{\mathbf{u}}f(2, 1) = f_x(2, 1)\left(-\frac{\sqrt{10}}{10}\right) + f_y(2, 1)\left(\frac{3\sqrt{10}}{10}\right) = -2\left(-\frac{\sqrt{10}}{10}\right) + 4\left(\frac{3\sqrt{10}}{10}\right) = \frac{7\sqrt{10}}{5}$.

29. Here $\mathbf{u} = \dfrac{\overrightarrow{PQ}}{\left|\overrightarrow{PQ}\right|} = \dfrac{\mathbf{i} + 3\mathbf{j}}{\sqrt{1^2 + 3^2}} = \frac{\sqrt{10}}{10}\mathbf{i} + \frac{3\sqrt{10}}{10}\mathbf{j}$, $f_x(x, y) = 3x^2$, and $f_y(x, y) = 3y^2$, so
$D_{\mathbf{u}}f(1, 2) = f_x(1, 2)\left(\frac{\sqrt{10}}{10}\right) + f_y(1, 2)\left(\frac{3\sqrt{10}}{10}\right) = 3\left(\frac{\sqrt{10}}{10}\right) + 12\left(\frac{3\sqrt{10}}{10}\right) = \frac{39\sqrt{10}}{10}$.