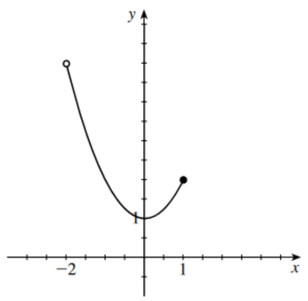
3.1 Extrema of Functions

12.



The absolute minimum value of h is 1, attained at $x = 0$.

32. $f'(x) = \frac{d}{dx}\left(\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x - 3\right) = x^2 + x + 2 \neq 0$ because the discriminant $1 - 4(1)(2) = -7 < 0$, and so $f'(x) = 0$ has no real root. Thus, f has no critical number.

47. $g'(x) = \frac{d}{dx}\left(3x^4 + 4x^3 + 1\right) = 12x^3 + 12x^2 = 12x^2(x+1)$, and so g has critical numbers -1 and 0 on the interval $(-2, 1)$. We calculate $g(-2) = 17$, $g(-1) = 0$, $g(0) = 1$, and $g(1) = 8$, so g has an absolute minimum value of 0 attained at $x = -1$ and an absolute maximum value of 17 attained at $x = -2$.

53. $f'(x) = \frac{d}{dx}\left(x - 2x^{1/2}\right) = 1 - x^{-1/2} = 1 - \frac{1}{x^{1/2}} = \frac{x^{1/2} - 1}{x^{1/2}}$ has 1 as a critical number in the interval $(0, 9)$. $f(0) = 0$, $f(1) = -1$, and $f(9) = 3$, so f has an absolute minimum value of -1 attained at $x = 1$ and an absolute maximum value of 3 attained at $x = 9$.

3.2 The Mean Value Theorem

10. $f(x) = x^3 - 2x^2 \Rightarrow f'(x) = 3x^2 - 4x$. f is continuous on $[-1, 2]$ and differentiable on $(-1, 2)$. Therefore, there exists a number c in the interval $(-1, 2)$ such that $\frac{f(2) - f(-1)}{2 - (-1)} = f'(c) \Rightarrow \frac{(8 - 8) - (-1 - 2)}{3} = 3c^2 - 4c \Leftrightarrow 1 = 3c^2 - 4c$ $\Leftrightarrow 3c^2 - 4c - 1 = 0$, giving $c = \frac{4 \pm \sqrt{16 - 4(3)(-1)}}{2(3)} = \frac{4 \pm \sqrt{28}}{6} = \frac{2 \pm \sqrt{7}}{3}$, both of which lie on $(-1, 2)$.

22. $f(x) = 1 - x^{2/3} \Rightarrow f'(x) = -\frac{2}{3}x^{-1/3} = -\frac{2}{3x^{1/3}}$. Suppose there is a number c satisfying $-1 < c < 8$ such that $f'(c) = \frac{f(b) - f(a)}{b - a} = \frac{f(8) - f(-1)}{8 - (-1)}$. Then $-\frac{2}{3c^{1/3}} = \frac{\left(1 - 8^{2/3}\right) - (1 - 1)}{9} = -\frac{1}{3} \Leftrightarrow c^{1/3} = 2 \Leftrightarrow c = 8$. This contradiction shows that no such c exists. The result does not contradict the Mean Value Theorem because f is not differentiable on the interval $(-1, 8)$.

24. $g(x) = x^4 - 2x^2 + x$ is continuous on $[0, 1]$ and differentiable on $(0, 1)$. Furthermore, $g(0) = 0 = g(1)$. Therefore, by Rolle's Theorem, there exists at least one number c in $(0, 1)$ such that $g'(c) = 4c^3 - 4c + 1 = 0$. But $g'(x) = f(x)$, and so $f(c) = 0$, showing that f has at least one zero in $(0, 1)$.

30. Let $g(x) = cx$. Then $g'(x) = c$ and so $g'(x) = f'(x)$. By Theorem 3, g and f must differ by a constant. Thus $f(x) = g(x) + d$, where d is constant, and so $f(x) = cx + d$ as desired.