聯微作業解答 (4.1, 4.2)

4.1

- Find the indefinite integral, and check your answer by differentiation.

9. $\int t^{\frac{1}{3}}(t-1)^2 dt$

Sol. $\int t^{\frac{1}{3}}(t-1)^2 dt = \int (t^{\frac{7}{3}} - 2t^{\frac{4}{3}} + t^{\frac{1}{3}})dt = \frac{3}{10}t^{\frac{10}{3}} - \frac{6}{7}t^{\frac{7}{3}} + \frac{3}{4}t^{\frac{4}{3}} + C,$

C is a constant.

$\frac{d}{dt}(\frac{3}{10}t^{\frac{10}{3}} - \frac{6}{7}t^{\frac{7}{3}} + \frac{3}{4}t^{\frac{4}{3}} + C) = t^{\frac{7}{3}} - 2t^{\frac{4}{3}} + t^{\frac{1}{3}}.$

23. $\int \frac{\cos x}{1-\cos^x}dx$

Sol. $\int \frac{\cos x}{1-\cos^2 x}dx = \int \frac{\cos x}{\sin^2 x}dx = \int \cot x \csc x dx = -\csc x + C,$

C is a constant.

$\frac{d}{dx}(-\csc x + C) = \cot x \csc x.$

27. $\int \frac{1}{\sin^2 x \cos^2 x}dx$

Hint: Use the identity $\sin^2 x + \cos^2 x = 1$.

Sol. $\int \frac{1}{\sin^2 x \cos^2 x}dx = \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}dx = \int (\sec^2 x + \csc^2 x)dx$

$= \tan x - \cot x + C$, C is a constant.

$\frac{d}{dx}(\tan x - \cot x + C) = \sec^2 x + \csc^2 x.$

- Find f by solving the initial value problem.

38. $f'(x) = 1 + \frac{1}{x^2}$, $f(1) = 2$

Sol. $f(x) = \int f'(x)dx = \int (1 + \frac{1}{x^2})dx = x - \frac{1}{x} + C,$

C is a constant.

$f(1) = 2 \Rightarrow 1 - 1 + C = 2 \Rightarrow C = 2.$

Thus $f(x) = x - \frac{1}{x} + 2.$

42. $f''(x) = 2x + 1$, $f(0) = 5$, f'(0)=1

Sol. $f'(x) = \int f''(x)dx = \int(2x+1)dx = x^2 + x + C_1$,

C_1 is constant.

$f'(0) = 1 \Rightarrow 0 + C_1 = 1 \Rightarrow C_1 = 1$. So $f'(x) = x^2 + x + 1$.

Then $f(x) = \int f'(x)dx = \int(x^2 + x + 1)dx = \frac{1}{3}x^3 + \frac{1}{2}x^2 + x + C_2$,

C_2 is constant.

$f(0) = 5 \Rightarrow C_2 = 5$. Thus $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 + x + 5$.

- Find the integral using the indicated substitution.

6. $\int \frac{\sin x}{\cos^2 x} dx$, $u = \cos x$

Sol. Let $u = \cos x$, $du = -\sin x dx$

$\int \frac{\sin x}{\cos^2 x} dx = -\int \frac{du}{u^2} = \frac{1}{u} + C = \frac{1}{\cos x} + C = \sec x + C$,

C is a constant.

- Find the indefinite integral.

14. $\int (x^2 - 1)(x^3 - 3x)^3 dx$

Sol. Let $u = x^3 - 3x$, $du = 3x^2 - 3dx = 3(x^2 - 1)dx$

$\int (x^2 - 1)(x^3 - 3x)^3 dx = \frac{1}{3} \int u^3 du = \frac{1}{12} u^4 + C = \frac{1}{12}(x^3 - 3x)^4 + C$,

C is a constant.

20. $\int \frac{x^2 + \frac{2}{3}}{(x^3 + 2x)^2} dx$

Sol. Let $u = x^3 + 2x$, $du = 3x^2 + 2dx = 3(x^2 + \frac{2}{3})dx$

$\int \frac{x^2 + \frac{2}{3}}{(x^3 + 2x)^2} dx = \frac{1}{3} \int \frac{1}{u^2} du = \frac{1}{3}(\frac{-1}{u}) + C = \frac{-1}{3x(x^2 + 2)} + C$

C is a constant.

32. $\int \cot^3 x \csc^2 x dx$

Sol. Let $u = \cot x$, $du = -\csc^2 x dx$

$\int \cot^3 x \csc^2 x dx = -\int u^3 du = -\frac{1}{4} u^4 + C = -\frac{1}{4} \cot^4 x + C$,

C is a constant.

34. $\int \sqrt{\sin\theta}\cos\theta d\theta$

Sol. Let $u = \sin\theta$, $du = \cos\theta d\theta$

$\int \sqrt{\sin\theta}\cos\theta d\theta = \int \sqrt{u} du = \frac{2}{3}u^{\frac{3}{2}} + C = \frac{2}{3}\sqrt{\sin^3\theta} + C$

C is a constant.