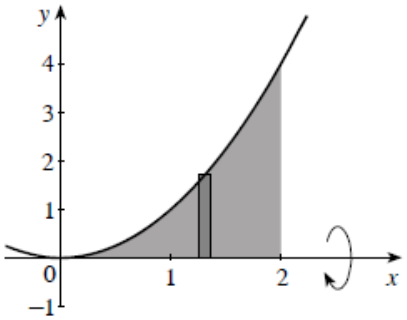
5.2

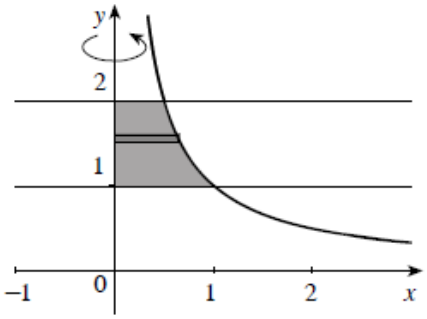
13. $V = \pi \int_0^2 y^2\,dx = \pi \int_0^2 \left(x^2\right)^2\,dx = \pi \int_0^2 x^4\,dx$

$= \pi \left(\frac{1}{5}x^5\right)\Big|_0^2 = \frac{32\pi}{5}$



17. $V = \pi \int_1^2 x^2\,dy = \pi \int_1^2 (1/y)^2\,dy = \pi \int_1^2 y^{-2}\,dy$

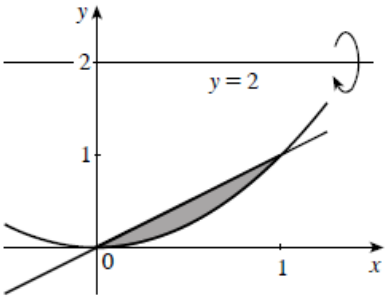
$= \pi\,(-1/y)\big|_1^2 = \frac{\pi}{2}$



34. $V = \pi \int_0^1 \left[\left(2 - x^2\right)^2 - (2 - x)^2\right] dx$

$= \pi \int_0^1 \left(x^4 - 5x^2 + 4x\right) dx$

$= \pi \left(\frac{1}{5}x^5 - \frac{5}{3}x^3 + 2x^2\right)\Big|_0^1 = \frac{8\pi}{15}$

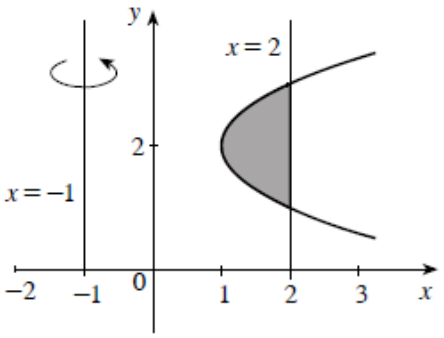


37. $V = \pi \int_1^3 \left\{(-1-2)^2 - \left[-1-\left(y^2-4y+5\right)\right]^2\right\} dy$

$= \pi \int_1^3 \left[9 - \left(y^2-4y+6\right)^2\right] dy$

$= \pi \int_1^3 \left(-y^4+8y^3-28y^2+48y-27\right) dy$

$= \pi \left(-\frac{1}{5}y^5+2y^4-\frac{28}{3}y^3+24y^2-27y\right)\Big|_1^3 = \frac{104\pi}{15}$



54. Here $A(x) = \frac{1}{2}\pi y^2 = \frac{1}{2}\pi\left(4-x^2\right)$, so

$V = \int_{-2}^2 A(x)\,dx = 2\int_0^2 A(x)\,dx = 2\int_0^2 \frac{1}{2}\pi\left(4-x^2\right) dx = \pi\int_0^2 \left(4-x^2\right) dx = \pi\left(4x-\frac{1}{3}x^3\right)\Big|_0^2 = \frac{16\pi}{3}.$

5.3

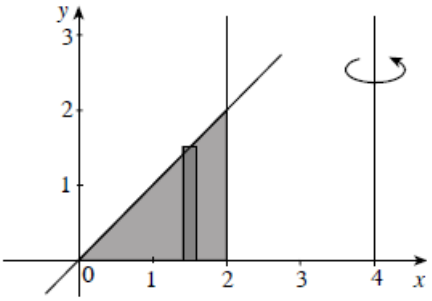
3. $V = 2\pi \int_0^1 x\left(x - x^2\right) dx = 2\pi \int_0^1 \left(x^2 - x^3\right) dx = 2\pi \left(\frac{1}{3}x^3 - \frac{1}{4}x^4\right)\Big|_0^1 = \frac{\pi}{6}$

6. $V = 2\pi \int_0^2 (3 - x)\left[\left(\frac{1}{2}x^2 + 2\right) - x^2\right] dx = 2\pi \int_0^2 \left(\frac{1}{2}x^3 - \frac{3}{2}x^2 - 2x + 6\right) dx = 2\pi \left(\frac{1}{8}x^4 - \frac{1}{2}x^3 - x^2 + 6x\right)\Big|_0^2 = 12\pi$

29. $V = 2\pi \int_0^2 (4 - x)\, x\, dx$

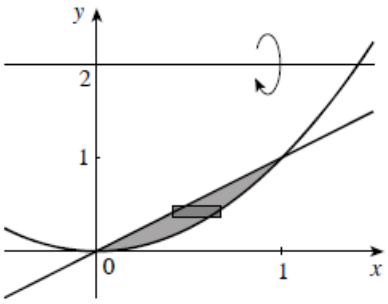
$= 2\pi \int_0^2 \left(4x - x^2\right) dx$

$= 2\pi \left(2x^2 - \frac{1}{3}x^3\right)\Big|_0^2 = \frac{32\pi}{3}$



34. $V = 2\pi \int_0^1 (2 - y)\left(\sqrt{y} - y\right) dy = 2\pi \int_0^1 \left(y^2 - y^{3/2} - 2y + 2y^{1/2}\right) dy$

$= 2\pi \left(\frac{1}{3}y^3 - \frac{2}{5}y^{5/2} - y^2 + \frac{4}{3}y^{3/2}\right)\Big|_0^1 = \frac{8\pi}{15}$



43. Using symmetry, we obtain

$V = 2 \cdot 2\pi \int_{-a}^{a} (b - x)\, y\, dx = 4\pi \int_{-a}^{a} (b - x)\left(a^2 - x^2\right)^{1/2} dx$

$= 4\pi b \int_{-a}^{a} \left(a^2 - x^2\right)^{1/2} dx - 4\pi \int_{-a}^{a} x\left(a^2 - x^2\right)^{1/2} dx$

Interpreting the integral geometrically, we find the first integral to be equal to $4\pi b\left(\frac{1}{2}\pi a^2\right)$. The second integral is 0 because the integrand is odd.

Thus, $V = 2\pi^2 a^2 b$.