

1. Evaluate the limit.

(a) (10%)  $\lim_{x \rightarrow \infty} (1 - \frac{1}{x})^x$ .

(b) (5%)  $\lim_{x \rightarrow 1} (\frac{1}{\ln x} - \frac{1}{x-1})$ .

2. (a) (8%) Find the integral  $\int \frac{\sqrt{\log_{10} x}}{x} dx$ .

(b) (7%) Find the derivative of  $y = (x + 2)^{1/x}$ .

3. (15%) Suppose that  $g$  is the inverse of a differentiable function  $f$  and  $H = g \circ g$ . If  $f(4) = 3$ ,  $g(4) = 5$ ,  $f'(4) = \frac{1}{2}$ , and  $f'(5) = 2$ , find  $H'(3)$ .

4. (15%) Let  $R$  be the region bounded by the graphs of the equations  $y = 4 - x^2$  and  $y = -x + 2$ . Use shell method to find the volume of the solid obtained by revolving  $R$  about the line  $x = 4$ .

5. (a) (5%) Find the integral  $\int \frac{1}{\sqrt{16-x^2}} dx$ .

(b) (5%) Find the derivative of  $f(x) = \tan^{-1} x^2$ .

6. (a) (5%) Evaluate  $\int_1^e \frac{\ln x}{x} e^{(\ln x)^2} dx$ .

(b) (5%) Differentiate the function  $h(x) = \tan(e^{2x} + \ln x)$ .

7. (10%) Find the volume of the solid obtained by revolving the region bounded by  $y = \sqrt{x}$  and  $y = x$  about the  $y$ -axis.

8. (a) (5%) Find the area of the region bounded by the graphs of  $y = 2 - x^2$  and  $y = -x$ .

(b) (5%) Find the area of the region bounded by the graphs of  $x = y^2$  and  $y = x - 2$ .