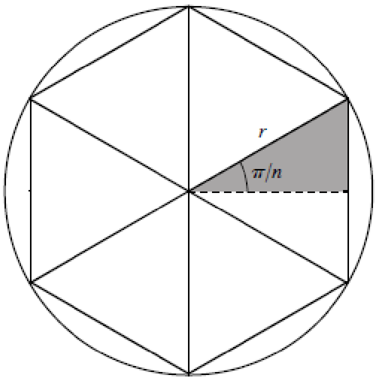
4.3

32. $\sum_{k=1}^{8}\left(3-k^2\right)=3\sum_{k=1}^{8}1-\sum_{k=1}^{8}k^2=3(8)-\dfrac{8(8+1)(2\cdot 8+1)}{6}=-180$

39. $\lim\limits_{n\to\infty}\sum_{k=1}^{n}\dfrac{2k}{n^2}=\lim\limits_{n\to\infty}\dfrac{1}{n^2}\cdot\dfrac{2n(n+1)}{2}=\lim\limits_{n\to\infty}\left(1+\dfrac{1}{n}\right)=1$

43.
$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(1+\frac{2k}{n}\right)^2\left(\frac{1}{n}\right)=\lim_{n\to\infty}\left(\frac{1}{n}\right)\left(\sum_{k=1}^{n}1+\frac{4}{n}\sum_{k=1}^{n}k+\frac{4}{n^2}\sum_{k=1}^{n}k^2\right)$$
$$=\lim_{n\to\infty}\left[\frac{1}{n}\cdot n+\frac{4}{n^2}\cdot\frac{n(n+1)}{2}+\frac{4}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}\right]$$
$$=\lim_{n\to\infty}\left[1+2\left(1+\frac{1}{n}\right)+\frac{2}{3}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)\right]=\frac{13}{3}$$

57. a. The area of the shaded triangle is $\frac{1}{2}$ (base) (height) $=\frac{1}{2}\left(r\cos\frac{\pi}{n}\right)\left(r\sin\frac{\pi}{n}\right)$, so the area of each isosceles triangle is $2\cdot\frac{1}{2}r^2\cos\frac{\pi}{n}\sin\frac{\pi}{n}=\frac{1}{2}r^2\sin\frac{2\pi}{n}$. Therefore, $A_n=\frac{1}{2}r^2 n\sin\frac{2\pi}{n}$.



b. $A=\lim\limits_{n\to\infty}A_n=\lim\limits_{n\to\infty}\frac{1}{2}r^2 n\sin\frac{2\pi}{n}=\frac{1}{2}r^2\lim\limits_{n\to\infty}n\left(\frac{2\pi}{2\pi}\right)\sin\frac{2\pi}{n}$

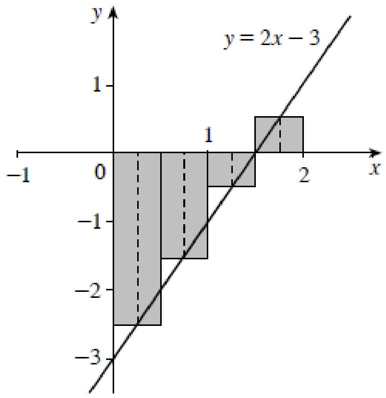
$=\frac{1}{2}r^2(2\pi)\lim\limits_{n\to\infty}\dfrac{\sin\frac{2\pi}{n}}{\frac{2\pi}{n}}=\pi r^2$

58. a. Refer to the figure in Exercise 57. The length of each side of the isosceles triangle is $2r\sin\frac{\pi}{n}$, so the perimeter of the polygon is $C_n=2nr\sin\frac{\pi}{n}$.

b. $C=\lim\limits_{n\to\infty}C_n=\lim\limits_{n\to\infty}2nr\sin\frac{\pi}{n}=2r\lim\limits_{n\to\infty}n\left(\frac{\pi}{\pi}\right)\sin\frac{\pi}{n}=2\pi r\lim\limits_{n\to\infty}\dfrac{\sin\frac{\pi}{n}}{\frac{\pi}{n}}=2\pi r$

4. 4

3. a.



b. $a = 0$, $b = 2$, and $n = 4$, so $\Delta x = \frac{2-0}{4} = \frac{1}{2}$, $x_0 = 0$, $x_1 = \frac{1}{2}$, $x_2 = 1$, $x_3 = \frac{3}{2}$, and $x_4 = 2$. Then $c_1 = \frac{1}{4}$, $c_2 = \frac{3}{4}$, $c_3 = \frac{5}{4}$, $c_4 = \frac{7}{4}$, and the Riemann sum is

$$\begin{aligned}\textstyle\sum_{k=1}^{4} f(c_k)\,\Delta x &= \left[f\left(\tfrac{1}{4}\right) + f\left(\tfrac{3}{4}\right) + f\left(\tfrac{5}{4}\right) + f\left(\tfrac{7}{4}\right)\right]\left(\tfrac{1}{2}\right) \\ &= \left\{\left[2\left(\tfrac{1}{4}\right) - 3\right] + \left[2\left(\tfrac{3}{4}\right) - 3\right] \right. \\ &\quad \left. + \left[2\left(\tfrac{5}{4}\right) - 3\right] + \left[2\left(\tfrac{7}{4}\right) - 3\right]\right\}\left(\tfrac{1}{2}\right) \\ &= -2.\end{aligned}$$

14. $\lim_{n\to\infty} \sum_{k=1}^{n} 2c_k\,(1 - c_k)^2\ \Delta x = \int_0^3 2x\,(1 - x)^2\ dx$

15. $\displaystyle\lim_{n\to\infty} \sum_{k=1}^{n} \frac{2c_k}{c_k^2 + 1}\,\Delta x = \int_1^2 \frac{2x}{x^2 + 1}\,dx$

47. Let $g(x) = -f(x)$. Then g is continuous and $g(x) \geq 0$ on $[a, b]$. By Property 4 of the definite integral, $0 \leq \int_a^b g(x)\,dx = \int_a^b [-f(x)]\,dx = -\int_a^b f(x)\,dx \Rightarrow \int_a^b f(x)\,dx \leq 0$.