Calculus II

Second Midterm

May 12, 2015

1. (15%) Find the radius of convergence and the interval of convergence of $\sum_{n=0}^{\infty} \frac{(-1)^n 2^n x^n}{\sqrt{n+1}}$.

2. (15%) Find the Taylor series for $f(x) = \ln x$ at 1, and determine its interval of convergence.

3. (15%) Find the equation of the plane containing the points $P(3, -1, 1)$, $Q(1, 4, 2)$, and $R(0, 1, 4)$.

4. (15%) Find the parametric equation for the tangent line to the curve $x = e^{-t}\cos t$, $y = e^{-t}\sin t$, $z = \sin^{-1} t$; at the point $t = 0$.

5. (10%) Write the vector $\mathbf{b} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ as the sum of a vector parallel to $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$ and a vector perpendicular to $\mathbf{a}$.

6. (10%) Find the volume of the parallelepiped determined by the vectors, $\mathbf{a} = \mathbf{i} + \mathbf{j}$, $\mathbf{b} = \mathbf{j} - 2\mathbf{k}$, $\mathbf{c} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$.

7. (10%) Find parametric equation for the line of intersection of the planes: $3x - y + 2z = 1$ and $2x + 3y - z = 4$.

8. (10%) Find the antiderivative of $\mathbf{r}'(t) = \cos t\mathbf{i} + e^{-t}\mathbf{j} + \sqrt{t}\mathbf{k}$ satisfying the initial condition $\mathbf{r}(0) = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$.