

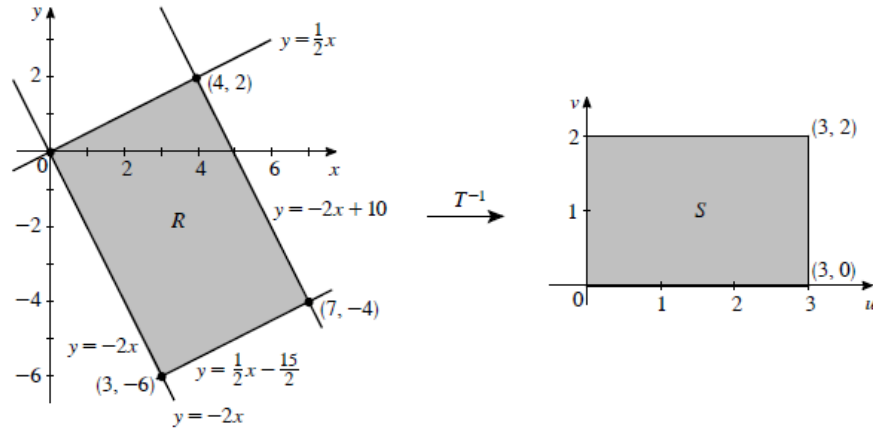
14.8 (不須繳交)

13. To find T^{-1} , we solve the system of equations of $T : x = u + 2v, y = v - 2u$ for u and v , obtaining

$T^{-1} : u = \frac{1}{5}(x - 2y), v = \frac{1}{5}(2x + y)$. Using this transformation, we obtain the region $S = T^{-1}(R)$. Next, we find

$$J = \frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 1 & 2 \\ -2 & 1 \end{vmatrix} = 5. \text{ Thus,}$$

$$\begin{aligned} \iint_R (x + y) dA &= \iint_S [(u + 2v) + (v - 2u)] \left| \frac{\partial(x, y)}{\partial(u, v)} \right| du dv = 5 \int_0^2 \int_0^3 (3v - u) du dv \\ &= 5 \int_0^2 \left[3uv - \frac{1}{2}u^2 \right]_{u=0}^{u=3} dv = 5 \int_0^2 \left(9v - \frac{9}{2} \right) dv = 5 \left(\frac{9}{2}v^2 - \frac{9}{2}v \right) \Big|_0^2 = 45 \end{aligned}$$

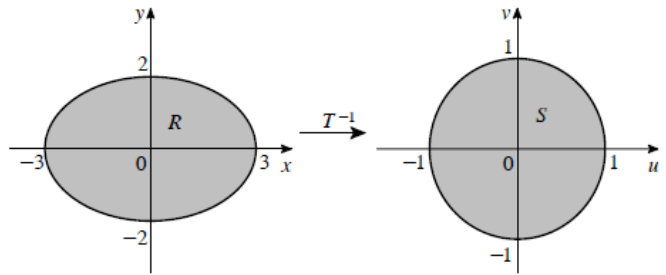


15. Here $T : x = 3u, y = 2v$. Then $4x^2 + 9y^2 = 36$

$\Rightarrow 4(3u)^2 + 9(2v)^2 = 36 \Leftrightarrow u^2 + v^2 = 1$, the circle with radius 1 centered at the origin of the uv -plane. Next, we find

$$J = \frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} 3 & 0 \\ 0 & 2 \end{vmatrix} = 6, \text{ so}$$

$$\begin{aligned} \iint_R 2xy dA &= 2 \iint_S (3u)(2v) \left| \frac{\partial(x, y)}{\partial(u, v)} \right| du dv = 12 \cdot 6 \int_0^{2\pi} \int_0^1 (r \cos \theta)(r \sin \theta)r dr d\theta = 72 \int_0^{2\pi} \left[\frac{1}{4}r^4 \cos \theta \sin \theta \right]_{r=0}^{r=1} d\theta \\ &= 72 \int_0^{2\pi} \frac{1}{4} \cos \theta \sin \theta d\theta = -9 \sin^2 \theta \Big|_0^{2\pi} = 0 \end{aligned}$$



21. Let $u = x + y$ and $v = 2x - y$. Then $-1 \leq u \leq 3$

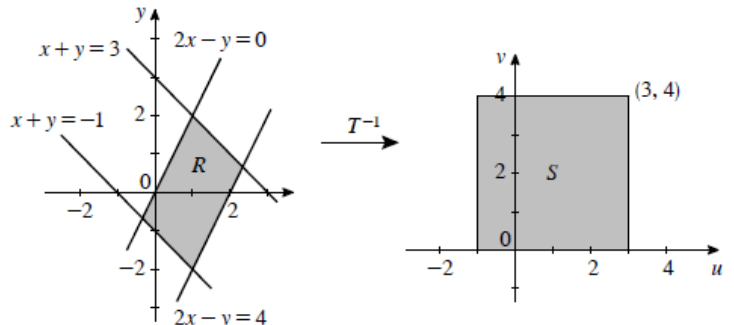
and $0 \leq v \leq 4$. Solving for x and y , we obtain

$x = \frac{1}{3}(u + v)$ and $y = \frac{1}{3}(2u - v)$. Thus,

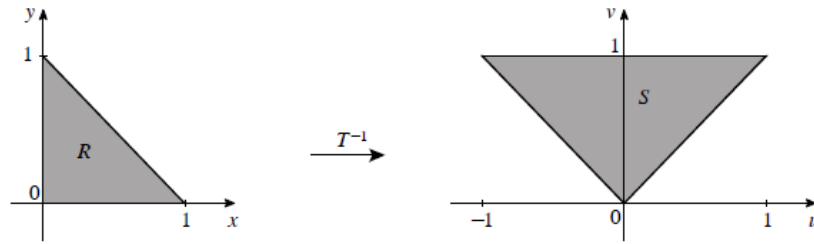
$$J = \frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{vmatrix} = -\frac{1}{3},$$

so

$$\begin{aligned} \iint_R (2x + y) dA &= \int_{-1}^3 \int_0^4 \left[\frac{2}{3}(u + v) + \frac{1}{3}(2u - v) \right] \left| -\frac{1}{3} \right| dv du = \frac{1}{9} \int_{-1}^3 \int_0^4 (4u + v) dv du \\ &= \frac{1}{9} \int_{-1}^3 \left[4uv + \frac{1}{2}v^2 \right]_{v=0}^{v=4} du = \frac{1}{9} \int_{-1}^3 (16u + 8) du = \frac{8}{9} (u^2 + u) \Big|_{-1}^3 = \frac{32}{3} \end{aligned}$$



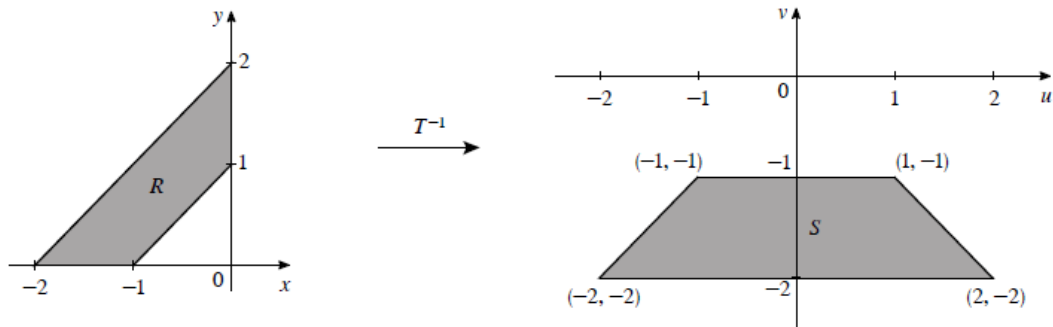
23. Let $T : u = x - y, v = x + y$. Then $T^{-1} : x = \frac{1}{2}(u + v), y = \frac{1}{2}(v - u)$. The triangular region R is mapped onto the triangular region S .



Here $J = \frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{vmatrix} = \frac{1}{2}$, so

$$\begin{aligned} \iint_R \exp\left(\frac{x-y}{x+y}\right) dA &= \int_0^1 \int_{-v}^v e^{u/v} \left|\frac{1}{2}\right| du dv = \frac{1}{2} \int_0^1 \left[ve^{u/v}\right]_{u=-v}^{u=v} dv = \frac{1}{2} \int_0^1 v(e - e^{-1}) dv = \frac{1}{2}(e - e^{-1}) \left(\frac{1}{2}v^2\right) \Big|_0^1 \\ &= \frac{e^2 - 1}{4e} \end{aligned}$$

24. Let $T : u = x + y, v = x - y$, so $T^{-1} : x = \frac{1}{2}(u + v), y = \frac{1}{2}(u - v)$. The trapezoidal region R is mapped onto the trapezoidal region S .



Now $J = \frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{vmatrix} = -\frac{1}{2}$, so

$$\begin{aligned} \iint_R \exp\left(\frac{x+y}{x-y}\right) dA &= \int_{-2}^{-1} \int_{-v}^v e^{u/v} \left|-\frac{1}{2}\right| du dv = \frac{1}{2} \int_{-2}^{-1} \left[ve^{u/v}\right]_{-v}^v dv = \frac{1}{2} \int_{-2}^{-1} (e^{-1} - e) v dv \\ &= \frac{1 - e^2}{2e} \left(\frac{v^2}{2}\right) \Big|_{-2}^{-1} = \frac{3(e^2 - 1)}{4e} \end{aligned}$$