# 9.6

**4.** $\sum_{n=1}^{\infty} \frac{(-2)^n}{n!}$. We use the Ratio Test on $a_n = \frac{(-1)^n 2^n}{n!}$, obtaining

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left[\frac{2^{n+1}}{(n+1)!}\cdot\frac{n!}{2^n}\right] = \lim_{n\to\infty}\frac{2}{n+1} = 0,$$ so the series converges absolutely.

**11.** $\sum_{n=1}^{\infty} \frac{n!}{e^n}$. Using the Ratio Test with $a_n = \frac{n!}{e^n}$, we have $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left[\frac{(n+1)!}{e^{n+1}}\cdot\frac{e^n}{n!}\right] = \lim_{n\to\infty}\frac{n+1}{e} = \infty$, so the series diverges.

**15.** $\sum_{n=1}^{\infty} \frac{2^n}{n!n}$. We use the Ratio Test: $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left[\frac{2^{(n+1)}}{(n+1)!\,(n+1)}\cdot\frac{n!n}{2^n}\right] = \lim_{n\to\infty}\frac{2n}{(n+1)^2} = 0$, so the series converges.

**19.** $\sum_{n=2}^{\infty} \frac{(-1)^n \ln n}{2^n}$. Consider $\sum_{n=2}^{\infty}\left|\frac{(-1)^n \ln n}{2^n}\right| = \sum_{n=2}^{\infty}\frac{\ln n}{2^n}$. Since $\frac{\ln n}{2^n} < \frac{n}{2^n}$ and $\sum_{n=2}^{\infty}\frac{n}{2^n}$ converges (see Exercise 9.3.28), the Comparison Test implies that the given series converges absolutely.

**24.** $\sum_{n=2}^{\infty}\left(\frac{\ln n}{n}\right)^n$. We use the Root Test: $\lim_{n\to\infty}\sqrt[n]{|a_n|} = \lim_{n\to\infty}\left[\left(\frac{\ln n}{n}\right)^n\right]^{1/n} = \lim_{n\to\infty}\frac{\ln n}{n} = 0$, so the series converges absolutely.