

考試注意事項：

1. 答案紙直行對折，兩直欄書寫作答。
2. 無清楚計算過程，不予計分。

試題：

1. (15%) Evaluate

$$\iint_R \frac{\sin x}{x} dA$$

where R is the region bounded by $y = 0$, $x = 1$, and $y = x$.

2. (15%) Evaluate

$$\iint_R \cos\left(\frac{x-y}{x+y}\right) dA$$

where R is the trapezoidal region with vertices $(1, 0)$, $(2, 0)$, $(0, 2)$ and $(0, 1)$.

3. (10%, 5%) Let $\mathbf{F} = 2xy\mathbf{i} + (1 + x^2 - y^2)\mathbf{j}$.
 - (a) Find a potential f such that $\mathbf{F} = \nabla f$.
 - (b) If F is a force field, find the work done by F in moving a particle along any path from $(1, 0)$ to $(2, 3)$.
4. (15%) Evaluate $\oint_C (y^2 + \tan x) dx + (x^3 + 2xy + \sqrt{y}) dy$, where C is the circle $x^2 + y^2 = 4$ and is oriented in a positive direction.
5. (10%) Find the surface area of the part of the paraboloid $z = 9 - x^2 - y^2$ that lies above the plane $z = 5$.
6. (10%) Evaluate $\iiint_T x dV$, where T is the part of the region in the first octant lying inside the sphere $x^2 + y^2 + z^2 = 1$.
7. (5%, 5%) Let $\mathbf{F} = xy\mathbf{i} + xz\mathbf{j} + xyz^2\mathbf{k}$. Find $\operatorname{div} \mathbf{F}$ and $\operatorname{curl} \mathbf{F}$.
8. (10%) Find the work done by the force field $\mathbf{F}(x, y, z) = -y\mathbf{i} + x\mathbf{j} + z\mathbf{k}$ in moving a particle along the helix C described by the parametric equations $x = \cos t$, $y = \sin t$, and $z = t$ from $(1, 0, 0)$ to $(0, 1, \frac{\pi}{2})$.