Calculus

考試注意事項：

1. 答案紙直行對折，兩直攔書寫作答。

2. 無清楚計算過程，不予計分。

試題：

1. (15%) Find the length of the arc of the helix C given by the vector function $\mathbf{r}(t) = 2\cos t\mathbf{i} + 2\sin t\mathbf{j} + t\mathbf{k}$, where $0 \le t \le 2\pi$.

2. (10%,5%) Let $z = f(x,y) = 2x^2 - xy$.

   (a) Find the differential dz.
   (b) Compute the value of dz if (x,y) changes from $(1,1)$ to $(0.98, 1.03)$.

3. (15%) Find the relative extrema of $f(x,y) = x^3 + y^2 - 2xy + 7x - 8y + 2$.

4. (15%) Find the absolute extreme value of $f(x,y) = 2x^2 + y^2 - 2y + 1$ subject to the constraint $x^2 + y^2 \le 4$.

5. Let $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j}$ and $\mathbf{b} = 2\mathbf{j} + \mathbf{k}$.

   (a) Find a unit vector $\mathbf{n}$ that is orthogonal to both $\mathbf{a}$ and $\mathbf{b}$.
   (b) Express $\mathbf{a} \times \mathbf{b}$ in terms of $|\mathbf{a} \times \mathbf{b}|$ and $\mathbf{n}$.

6. (5%,5%) Use the Chain Rule to find the indicated derivative:

$$w = \cos(2x + 3y), \quad x = r^2 st, \quad y = s^2 tu; \quad \frac{\partial w}{\partial r} \text{ and } \frac{\partial w}{\partial u}$$

7. (5%,5%)

   (a) Find the directional derivative of $f(x,y) = 4 - 2x^2 - y^2$ at the point $(1,1)$ in the direction of the unit vector $\mathbf{u}$ that makes an angle of $\pi/3$ radians with positive x-axis.
   (b) Find the directional derivative of $f(x,y) = e^x \cos(2y)$ at the point $(0, \frac{\pi}{4})$ in the direction of $\mathbf{v} = 2\mathbf{i} + 3\mathbf{j}$.

8. (5%,5%) Find the equations of the tangent plane and normal line to the ellipsoid with equation $4x^2 + y^2 + 4z^2 = 16$ at $(1, 2, \sqrt{2})$.