Calculus

2012/03/27

考試注意事項：

1. 答案紙直行對折，兩直欄書寫作答。
2. 無清楚計算過程，不予計分。

試題：

1. (5%,10%) Find the radius of convergence and the interval of convergence of $\sum_{n=0}^{\infty} \frac{(-1)^n 2^n x^n}{\sqrt{n+1}}$.
2. (10%,5%) Find the Maclaurin series of $f(x) = \sin x$, and determine its interval of convergence.
3. (10%,5%) Let $f(x) = \sqrt{x}$.
   - Find the Taylor polynomial $P_2(x)$ of degree 2 at $c = 4$.
   - What is the maximum error incurred if f approximated by $P_2(x)$ on the interval $[3, 5]$?
4. (15%) Find the length of one arch of the cycloid
$$x = a(\theta - \sin\theta) \quad y = a(1 - \cos\theta).$$
5. (10%) Consider the cardioid $r = 1+\cos\theta$. Find the slope of the tangent line to the cardioid at the point where $\theta = \pi/6$.
6. (5% x 6) Determine whether the series is convergent or divergent. Indicate the tests that you use.
   (a) $\sum_{n=1}^{\infty} \frac{2n-1}{3n+1}$
   (b) $\sum_{n=1}^{\infty} \left[\frac{2}{3^n} - \frac{1}{n(n+1)}\right]$
   (c) $\sum_{n=3}^{\infty} \frac{1}{n\sqrt{\ln n}}$
   (d) $\sum_{n=3}^{\infty} \frac{\ln n}{n^2}$
   (e) $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{n^2+1}$
   (f) $\sum_{n=1}^{\infty} \frac{\sin n}{\sqrt{n^3+1}}$